SU PRUEBA DE PRÁCTICA

ANÁLISIS Y ENFOQUES

NIVEL SUPERIOR
PARA LAS MATEMÁTICAS DEL PD DEL IB

ANSWERS

Stephen Lee
Michael Cheung

- 4 Sets de Pruebas de Práctica
- Distribución de los Temas del Examen
- Análisis del Formato de Examen
- Lista Exhaustivas de Fórmulas

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Solución de Práctica de Prueba 1 de AE NS Set 1

Sección A

1. (a) La media

$= \frac{300}{15}$ (M1) por enfoque válido

$= 20$ A1

[2]

(b) (i) -40 A1

(ii) La varianza nueva

$= (-2)^2(9)$ (M1) por enfoque válido

$= 36$ A1

(iii) 6 A1

[4]

2. (a) La pendiente de L_1

$= \frac{32-0}{24-8}$ (M1) por enfoque válido

$= 2$

La ecuación de L_1 :

$y - 0 = 2(x-8)$ A1

$y = 2x - 16$

$2x - y - 16 = 0$ A1

[3]

(b) $2 \times -\frac{1}{-a} = -1$ (M1) por enfoque válido

$2 = -a$

$a = -2$ A1

[2]

3. (a) Lado izquierdo

$=(2n+1)^2+(2n+3)^2+(2n+5)^2$

$=4n^2+4n+1+4n^2+12n+9+4n^2+20n+25$ M1A1

$=12n^2+36n+35$

$=12n^2+36n+33+2$ M1

$=3(4n^2+12n+11)+2$

$=$Lado derecho AG

[3]

(b) $2n+1$, $2n+3$ y $2n+5$ son tres números impares consecutivos. R1

$(2n+1)^2+(2n+3)^2+(2n+5)^2$

$=3(4n^2+12n+11)+2$ A1

Además $3(4n^2+12n+11)$ es múltiplo de 3. R1

Por tanto, la suma de los cuadrados de tres números impares consecutivos es mayor que un múltiplo de 3 en 2 unidades. AG

[3]

4. $f(x)=px^3+qx^2-2x+1$

$f'(x)=p(3x^2)+q(2x)-2(1)+0$ (A1) por derivadas correctas

$f'(x)=3px^2+2qx-2$

$f'(1)=-1\div-\frac{1}{15}$

$\therefore 3p(1)^2+2q(1)-2=15$ (M1) por ecuación

$3p+2q=17$

$2q=17-3p$ A1

$f^{-1}(41)=2$

$\therefore f(2)=41$ (M1) por enfoque válido

$p(2)^3+q(2)^2-2(2)+1=41$ A1

$8p+4q-3=41$

$\therefore 8p+2(17-3p)-3=41$ (M1) por sustitución

$8p+34-6p-3=41$

$2p=10$

$p=5$ A1

$\therefore q=\frac{17-3(5)}{2}$

$q=1$ A1

[8]

5. (a) $a = \frac{37-(-5)}{2}$ M1

$a = 21$ A1

$b = \frac{2\pi}{2(11-2)}$ M1

$b = \frac{\pi}{9}$ A1

$d = \frac{37+(-5)}{2}$ M1

$d = 16$

$\therefore f(t) = 21\operatorname{sen}\frac{\pi}{9}(t+2,5)+16$ AG

[5]

(b) Las coordenadas de P'

$= (3(2)+17, 37+8)$ A1

$= (23, 45)$ A1

[2]

6. (a) $g(x)$

$= 3f(x-1)$

$= 3(4(x-1)^4 + 3(x-1)^2 - 1)$ (A1) por sustitución

$= 3(4(x^4 - 4x^3 + 6x^2 - 4x + 1) + 3(x^2 - 2x + 1) - 1)$ M1A1

$= 3(4x^4 - 16x^3 + 24x^2 - 16x + 4 + 3x^2 - 6x + 3 - 1)$ M1

$= 3(4x^4 - 16x^3 + 27x^2 - 22x + 6)$

$= 12x^4 - 48x^3 + 81x^2 - 66x + 18$ A1

[5]

(b) La suma de las raíces

$= -\frac{-48}{12}$ M1

$= 4$ A1

[2]

7. $1+f(|x|)\le |x|$

$$1+\frac{2|x|^3-5|x|^2-37}{|x|+37}\le |x|$$ M1

$$\frac{2|x|^3-5|x|^2-37}{|x|+37}\le |x|-1$$

$2|x|^3-5|x|^2-37\le (|x|-1)(|x|+37)$

$2|x|^3-5|x|^2-37\le |x|^2+36|x|-37$

$2|x|^3-6|x|^2-36|x|\le 0$ (A1) por inecuación correcta

$2|x|\left(|x|^2-3|x|-18\right)\le 0$

$|x|^2-3|x|-18\le 0$ M1

$(|x|+3)(|x|-6)\le 0$

$\therefore 0\le |x|\le 6$ A1

$\therefore 1< x\le 6$ A1

[5]

8. Cuando $n = 2$,

Lado izquierdo $= \binom{2}{2}$

Lado izquierdo $= 1$

Lado derecho $= \dfrac{2(2+1)(2-1)}{6}$

Lado derecho $= 1$

Por tanto, el enunciado es verdadero cuando $n = 2$. R1

Suponemos que es verdadero cuando $n = k$. M1

$$\binom{2}{2} + \binom{3}{2} + \ldots + \binom{k}{2} = \frac{k(k+1)(k-1)}{6}$$

Cuando $n = k+1$,

$$\binom{2}{2} + \binom{3}{2} + \ldots + \binom{k}{2} + \binom{k+1}{2}$$

$$= \frac{k(k+1)(k-1)}{6} + \binom{k+1}{2}$$ M1A1

$$= \frac{k(k+1)(k-1)}{6} + \frac{(k+1)(k)}{2}$$ A1

$$= \frac{k(k+1)(k-1)}{6} + \frac{3k(k+1)}{6}$$

$$= \frac{k(k+1)}{6}(k-1+3)$$

$$= \frac{k(k+1)(k+2)}{6}$$

$$= \frac{(k+1)((k+1)+1)((k+1)-1)}{6}$$ A1

Por tanto, el enunciado es verdadero cuando $n = k+1$.

A partir de lo anterior, el enunciado es verdadero para $n \in \mathbb{Z}^+$, $n \geq 2$. R1

[7]

9. (a) 1 A1

[1]

(b) $\int_1^a \frac{1}{e^2-1}e^{3-x}\,\mathrm{d}x = \frac{1}{2}$ A1

$\left[-\frac{1}{e^2-1}e^{3-x}\right]_1^a = \frac{1}{2}$ A1

$-\frac{1}{e^2-1}e^{3-a} - \left(-\frac{1}{e^2-1}e^2\right) = \frac{1}{2}$

$\frac{-e^{3-a}+e^2}{e^2-1} = \frac{1}{2}$ M1

$-e^{3-a}+e^2 = \frac{1}{2}e^2 - \frac{1}{2}$

$e^{3-a} = \frac{e^2+1}{2}$ A1

$3-a = \ln\left(\frac{e^2+1}{2}\right)$

$a = 3-\ln\left(\frac{e^2+1}{2}\right)$

Por tanto, la mediana es $3-\ln\left(\frac{e^2+1}{2}\right)$. AG

[4]

Sección B

10. (a) (i) $\{y : 0 \le y \le 1,\ y \in \mathbb{R}\}$ A2

(ii) $f(x) = 1$

$\therefore \cos^4 x = 1$

$\cos^2 x = -1$ (*Rechazada*) o $\cos^2 x = 1$ (M1) por enfoque válido

$\cos x = -1$ o $\cos x = 1$

$x = \pi$ o $x = 0$, $x = 2\pi$ (A1) por valores correctos

Por lo tanto, hay 3 soluciones. A1

[5]

(b) $f'(x) = (4\cos^3 x)(-\operatorname{sen} x)$ (A1) por regla de la cadena

$f'(x) = -4\operatorname{sen} x \cos^3 x$ A1

[2]

(c) El área total de las regiones

$= \int_0^{\pi} (\cos^4 x)(2\operatorname{sen} x)\mathrm{d}x$ (A1) por integral definida

Sea $u = \cos x$
$\dfrac{\mathrm{d}u}{\mathrm{d}x} = -\operatorname{sen} x \Rightarrow (-1)\mathrm{d}u = \operatorname{sen} x\mathrm{d}x$
$x = \pi \Rightarrow u = \cos \pi = -1$
$x = 0 \Rightarrow u = \cos 0 = 1$

(A1) por sustitución

$= \int_1^{-1} -2u^4 \mathrm{d}u$ M1A1

$= \left[-\frac{2}{5}u^5\right]_1^{-1}$ A1

$= -\frac{2}{5}(-1)^5 - \left(-\frac{2}{5}(1)^5\right)$ (M1) por sustitución

$= \frac{4}{5}$ A1

[7]

11. (a) $\frac{dy}{dx} = h(x)\cdot(y+1)$

$\frac{1}{y+1}dy = \text{sen } x dx$ (M1) por enfoque válido

$\int \frac{1}{y+1}dy = \int \text{sen } x dx$ (A1) por enfoque correcto

$\ln|y+1| = -\cos x + C$ A1

$y+1 = e^{-\cos x + C}$ (M1) por enfoque válido

$y = e^{-\cos x + C} - 1$ A1

$0 = e^{-\cos 0 + C} - 1$ (M1) por sustitución

$1 = e^{-1+C}$

$-1 + C = 0$

$C = 1$ (A1) por valor correcto

$\therefore y = e^{1-\cos x} - 1$ A1

[8]

(b) $\frac{dy}{dx} = h(x)\sqrt{1-(h(x))^2}\cdot(y+1)$

$\frac{dy}{dx} = \text{sen } x\sqrt{1-\text{sen}^2 x}\cdot(y+1)$

$\frac{dy}{dx} = \text{sen } x \cos x \cdot (y+1)$ A1

$\frac{dy}{dx} = \frac{\text{sen } 2x \cdot (y+1)}{2}$

$\frac{dy}{dx} - \left(\frac{1}{2}\text{sen } 2x\right)y = \frac{1}{2}\text{sen } 2x$ A1

El factor de integración

$= e^{\int -\frac{1}{2}\text{sen } 2x dx}$ M1

$= e^{\frac{1}{4}\cos 2x}$ A1

$\therefore e^{\frac{1}{4}\cos 2x}\frac{dy}{dx} - e^{\frac{1}{4}\cos 2x}\left(\frac{1}{2}\text{sen } 2x\right)y$

$= e^{\frac{1}{4}\cos 2x}\left(\frac{1}{2}\text{sen } 2x\right)$ M1

$\therefore e^{\frac{1}{4}\cos 2x}\frac{dy}{dx} - \frac{1}{2}ye^{\frac{1}{4}\cos 2x}\text{ sen } 2x = \frac{1}{2}e^{\frac{1}{4}\cos 2x}\text{ sen } 2x$

$\frac{d}{dx}\left(ye^{\frac{1}{4}\cos 2x}\right) = \frac{1}{2}e^{\frac{1}{4}\cos 2x}\text{ sen } 2x$ A1

$ye^{\frac{1}{4}\cos 2x} = \int \frac{1}{2}e^{\frac{1}{4}\cos 2x}\text{ sen } 2x dx$

Sea $u = \frac{1}{4}\cos 2x$. M1

$$\frac{du}{dx} = \frac{1}{4}(-\operatorname{sen} 2x)(2) \Rightarrow (-1)du = \frac{1}{2}\operatorname{sen} 2x dx$$

$\therefore ye^{\frac{1}{4}\cos 2x} = \int -e^u du$ A1

$ye^{\frac{1}{4}\cos 2x} = -e^u + C$

$ye^{\frac{1}{4}\cos 2x} = -e^{\frac{1}{4}\cos 2x} + C$

$y = Ce^{-\frac{1}{4}\cos 2x} - 1$ A1

$0 = Ce^{-\frac{1}{4}\cos 2(0)} - 1$ M1

$1 = Ce^{-\frac{1}{4}}$

$C = e^{\frac{1}{4}}$ A1

$\therefore y = e^{\frac{1}{4}-\frac{1}{4}\cos 2x} - 1$

$y = e^{\frac{1}{4}-\frac{1}{4}(1-2\operatorname{sen}^2 x)} - 1$ A1

$y = e^{\frac{1}{2}\operatorname{sen}^2 x} - 1$ AG

[12]

12. (a) $z^6+1=0$

$z^6=-1$

$z^6=\cos\pi+\mathrm{i}\,\mathrm{sen}\,\pi$ A1

$z=\cos\left(\frac{\pi+2k\pi}{6}\right)+\mathrm{i}\,\mathrm{sen}\left(\frac{\pi+2k\pi}{6}\right)$ M1

($k=0,1,2,3,4,5$)

$z=\cos\frac{\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{\pi}{6}$, $z=\cos\frac{\pi}{2}+\mathrm{i}\,\mathrm{sen}\frac{\pi}{2}$,

$z=\cos\frac{5\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{5\pi}{6}$, $z=\cos\frac{7\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{7\pi}{6}$,

$z=\cos\frac{3\pi}{2}+\mathrm{i}\,\mathrm{sen}\frac{3\pi}{2}$ o $z=\cos\frac{11\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{11\pi}{6}$ A2

[4]

(b) z^6+1

$=z^6-z^4+z^2+z^4-z^2+1$ M1

$=z^2(z^4-z^2+1)+(z^4-z^2+1)$

$=(z^2+1)(z^4-z^2+1)$ A1

$z^4-z^2+1=0$

$\frac{z^6+1}{z^2+1}=0$, donde $z^2\neq-1$ M1

$z=\cos\frac{\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{\pi}{6}$,

$z=\cos\frac{\pi}{2}+\mathrm{i}\,\mathrm{sen}\frac{\pi}{2}$ (*Rechazada*),

$z=\cos\frac{5\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{5\pi}{6}$, $z=\cos\frac{7\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{7\pi}{6}$,

$z=\cos\frac{3\pi}{2}+\mathrm{i}\,\mathrm{sen}\frac{3\pi}{2}$ (*Rechazada*) o

$z=\cos\frac{11\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{11\pi}{6}$ A1

[4]

(c) (i) $(z-p)(z-q)=0$ M1

$z^2-(p+q)z+pq=0$

$p+q=\lambda^3+\lambda+\lambda^{11}+\lambda^9$

$p+q=\lambda^3+\lambda+\lambda^5(-1)+\lambda^3(-1)$ M1

$p+q=\lambda^3+\lambda-\lambda^5-\lambda^3$

$p+q=\lambda-\lambda^5$

$p+q=\lambda-\frac{-1}{\lambda}$ M1

$p+q=\lambda+\frac{1}{\lambda}$

$\therefore p+q=\sqrt{3}$ A1

$pq=(\lambda^3+\lambda)(\lambda^{11}+\lambda^9)$

$pq=\lambda^{14}+\lambda^{12}+\lambda^{12}+\lambda^{10}$ M1

$pq=\lambda^2(1)+1+1+\lambda^4(-1)$ M1

$pq=\lambda^2-\lambda^4+2$

$pq=\lambda^2-(\lambda^2-1)+2$

$pq=3$ A1

$\therefore z^2-\sqrt{3}z+3=0$ A1

(ii) $(z-(2p))(z-(2q))=0$ M1

$z^2-(2p+2q)z+(2p)(2q)=0$

$z^2-2(p+q)z+4pq=0$ A1

$z^2-2\sqrt{3}z+4(3)=0$ M1

$z^2-2\sqrt{3}z+12=0$ A1

[12]

Solución de Práctica de Prueba 2 de AE NS Set 1

Sección A

1. (a) $y = 3x + 7$

$\Rightarrow x = 3y + 7$ (A1) por enfoque correcto

$3y = x - 7$

$y = \frac{x-7}{3}$

$\therefore f^{-1}(x) = \frac{x-7}{3}$ A1

[2]

(b) $(f \circ g)(x)$

$= 3g(x) + 7$ (A1) por sustitución

$= 3(2\sqrt{x}) + 7$

$= 6\sqrt{x} + 7$ A1

[2]

(c) $(f \circ g)(529)$

$= 6\sqrt{529} + 7$ (M1) por sustitución

$= 145$ A1

[2]

2. (a) El volumen

$=\frac{1}{3}\pi r^2 h$ (M1) por enfoque válido

$=\frac{1}{3}\pi(18)^2(18)$

$=6107,256119$ (A1) por valor correcto

$=6110$

$=6,11\times10^3 \text{ cm}^3$ A1

[3]

(b) $V=27\left(\frac{2}{3}\pi R^3\right)$ (M1) por inecuación

$16(6107,256119)=18\pi R^3$ (A1) por sustitución

$R^3=1728$

$R=12$ A1

La razón

$=18:12$

$=3:2$ A1

[4]

3. (a) $r=\frac{5,4}{4,5}$ (M1) por enfoque válido

$r=1,2$ A1

[2]

(b) $S_{12}=\frac{4.5(1,2^{12}-1)}{1,2-1}$ (A1) por sustitución

$S_{12}=178,1122601$

$S_{12}=178$ A1

[2]

(c) $u_n<678$

$4,5\cdot1,2^{n-1}<678$

$4,5\cdot1,2^{n-1}-678<0$ (M1) por enfoque válido

Considerando el gráfico de $y=4,5\cdot1,2^{n-1}-678$,

$n<28,50673$. A1

Por lo tanto, el mayor valor de n es 28. A1

[3]

4. (a) $20P_1 - 17P_0 = 0$

$\therefore 20(P_0 e^{k(1)}) - 17P_0 = 0$ A1

$20e^k - 17 = 0$

$e^k = 0,85$ M1

$k = \ln 0,85$ AG

[2]

(b) $\dfrac{P_t}{P_0} \le 0,5$

$\therefore \dfrac{P_0 e^{(\ln 0,85)t}}{P_0} \le 0,5$ (A1) por inecuación correcta

$e^{(\ln 0,85)t} \le 0,5$ (A1) por enfoque correcto

$(\ln 0,85)t \le \ln 0,5$

$(\ln 0,85)t - \ln 0,5 \le 0$ A1

Considerando el gráfico de

$y = (\ln 0,85)t - \ln 0,5$, $t \ge 4,2650243$. (M1) por enfoque válido

Por lo tanto, el menor número de años completos es 43. A1

[5]

5. (a) $\mathrm{AB}^2 = r^2 + r^2 - 2(r)(r)\cos 2\alpha$ A1

$\mathrm{AB}^2 = 2r^2 - 2r^2 \cos 2\alpha$

$\mathrm{AB} = \sqrt{2r^2 - 2r^2 \cos 2\alpha}$ A1

$\mathrm{AB} = \sqrt{2r^2(1 - \cos 2\alpha)}$

$\mathrm{AB} = r\sqrt{2(1 - \cos 2\alpha)}$ AG

[2]

(b) La longitud del arco ACB

$= (r)(2\alpha)$ A1

$= 2r\alpha$

$\therefore P$

$= 2r\alpha + r\sqrt{2(1 - \cos 2\alpha)}$ M1

$= 2r\alpha + r\sqrt{2(1 - (1 - 2\operatorname{sen}^2\alpha))}$ A1

$= 2r\alpha + r\sqrt{2(2\operatorname{sen}^2\alpha)}$ A1

$= 2r\alpha + r\sqrt{4\operatorname{sen}^2\alpha}$

$= 2r\alpha + 2r\operatorname{sen}\alpha$ A1

$= 2r(\alpha + \operatorname{sen}\alpha)$ AG

[5]

6. Por utilizar operaciones de fila ,el sistema

$$\begin{pmatrix} 1 & 3 & 2 & | & 1 \\ 5 & 7 & 1 & | & 2 \\ 32 & 24 & -17 & | & 5 \end{pmatrix} \text{ se reduce a } \begin{pmatrix} 1 & 0 & -\frac{11}{8} & | & -\frac{1}{8} \\ 0 & 1 & \frac{9}{8} & | & \frac{3}{8} \\ 0 & 0 & 0 & | & 0 \end{pmatrix}.$$ (M1) por enfoque válido

$y+\frac{9}{8}z=\frac{3}{8}$

$y=\frac{3}{8}-\frac{9}{8}z$ A1

$x-\frac{11}{8}z=-\frac{1}{8}$

$x=-\frac{1}{8}+\frac{11}{8}z$ A1

Sea $z=t$.

$x=-\frac{1}{8}+\frac{11}{8}t$, $y=\frac{3}{8}-\frac{9}{8}t$

Por lo tanto, la ecuación vectorial de la recta de intersección es $\mathbf{r}=\begin{pmatrix} -\frac{1}{8} \\ \frac{3}{8} \\ 0 \end{pmatrix}+t\begin{pmatrix} \frac{11}{8} \\ -\frac{9}{8} \\ 1 \end{pmatrix}$. A2

[5]

7. (a) $\dfrac{1-x}{1+ax}$

$= (1-x)(1+ax)^{-1}$

$= (1-x)\left(1+(-1)(ax)+\dfrac{(-1)(-2)}{2!}(ax)^2+\cdots\right)$ M1A1

$= (1-x)(1-ax+a^2x^2+\cdots)$

$= 1-ax+a^2x^2-x+ax^2-a^2x^3+\cdots$

$= 1+(-a-1)x+(a^2+a)x^2+\cdots$ A1

[3]

(b) (i) $1+(a^2+a)=21$ (A1) por ecuación correcta

$a^2+a-20=0$

$(a+5)(a-4)=0$

$a=-5$ (*Rechazado*) o $a=4$ A1

(ii) -5 A1

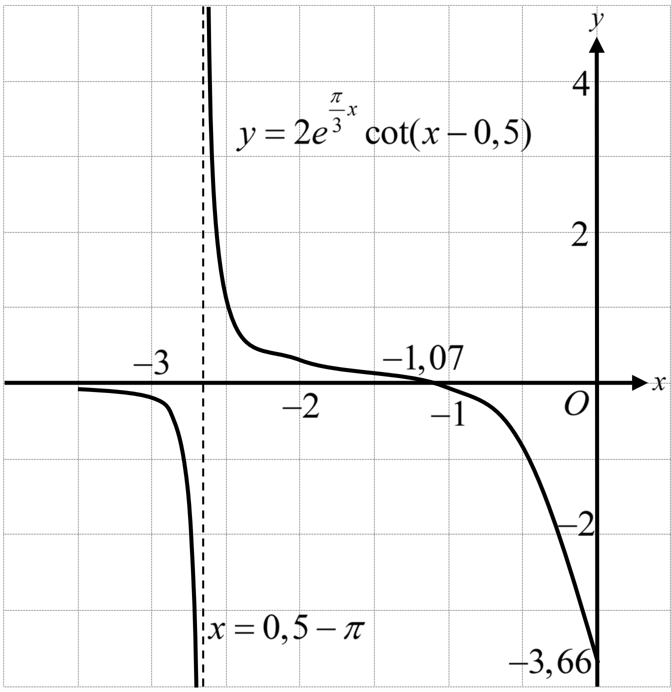
[3]

8. (a) Por forma correcta A1

Por asíntota correcta A1

Por puntos de corte correctos A1

[3]



(b) $0,0442 \le k \le 3,66$ A2

[2]

9. $y = 4^{-x}$

$\log_4 y = -x$

$x = -\log_4 y$ (A1) por enfoque correcto

$y = 4^{-0}$

$y = 1$ (A1) por valor correcto

$-\log_4 y = -\frac{1}{32}(y-24)^2$ (M1) por inecuación

$\frac{1}{32}(y-24)^2 - \log_4 y = 0$

Considerando el gráfico de $x = \frac{1}{32}(y-24)^2 - \log_4 y$,

$y = 16$. (A1) por valor correcto

$0 = -\frac{1}{32}(y-24)^2$

$0 = (y-24)^2$

$y = 24$ (A1) por valor correcto

El área de R

$= -\int_1^{16}(-\log_4 y)\mathrm{d}y - \int_{16}^{24} -\frac{1}{32}(y-24)^2\,\mathrm{d}y$ A1

$= 26,51312053$

$= 26,5$ A1

[7]

Sección B

10. (a) La probabilidad requerida

$= \mathrm{P}(T \leq 24)$	(M1) por enfoque válido
$= 0,9452007106$	
$= 0,945$	A1

[2]

(b)

$\mathrm{P}(U \leq 48) = 0,99494$	
$\mathrm{P}\left(Z \leq \dfrac{48-\mu}{7}\right) = 0,99494$	(M1) por estandarización
$\dfrac{48-\mu}{7} = 2,571701859$	A1
$48-\mu = 18,00191301$	
$\mu = 29,99808699$	
$\mu = 30,0$	A1

[3]

(c) La probabilidad requerida

$= \mathrm{P}(U \leq 36)$	R1
$= 0,8043925789$	A1
Por lo tanto, de todos los autobuses escolares que salen a las $8:24\ \mathrm{am}$, $80,439\%$ de ellos llegan al clase a tiempo.	AG

[2]

(d) La probabilidad requerida

$= 1 - \mathrm{P}(T \leq 12)\mathrm{P}(U \leq 48)$ $-\mathrm{P}(12 < T \leq 24)\mathrm{P}(U \leq 36)$	M1A1
$= 1 - (0,2118553337)(0,99494)$ $-(0,7333453769)(0,80439)$	(A2) por valores correctos
$= 0,1993209666$	
$= 0,199$	A1

[5]

(e) El número esperado

$= (20)(0,1993209666)$	(A1) por fórmula correcta
$= 3,986419331$	
$= 3,99$	A1

[2]

11. (a) Sean $\mathbf{n}_1$ y $\mathbf{n}_2$ los vectores normales a los planos π_1 y π_2 respectivamente.

$\mathbf{n}_1 = \begin{pmatrix} 4 \\ 3 \\ k \end{pmatrix}$, $\mathbf{n}_2 = \begin{pmatrix} 4 \\ -3 \\ k \end{pmatrix}$ (A1) por valores correctos

$$\mathbf{n}_1 \times \mathbf{n}_2 = \beta \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

$\therefore \begin{pmatrix} (3)(k)-(k)(-3) \\ (k)(4)-(4)(k) \\ (4)(-3)-(3)(4) \end{pmatrix} = \beta \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$, donde β es una constante. (A1) por sustitución

$$\begin{pmatrix} 6k \\ 0 \\ -24 \end{pmatrix} = \beta \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

$\therefore \frac{6k}{-24} = \frac{-3}{1}$ A1

$k = 12$ A1

[4]

(b) (i) $a = 6$, $b = 8$, $c = 2$, $\alpha = -6$ A4

(ii) Sea O el origen.
El volumen de la piramide $\mathrm{A'ABC}$

$= \frac{1}{3}\left(\frac{(\mathrm{A'A})(\mathrm{OB})}{2}\right)(\mathrm{OC})$ (M1) por enfoque válido

$= \frac{1}{3}\left(\frac{(6-(-6))(8)}{2}\right)(2)$ A1

$= 32$ A1

[7]

(c) (i) $\overrightarrow{\mathrm{AC}'} = -6\mathbf{i} - 2\mathbf{k}$

$\overrightarrow{\mathrm{AC}'} \cdot (-\mathbf{i}) = \left|\overrightarrow{\mathrm{AC}'}\right| \left|-\mathbf{i}\right| \cos \mathrm{C}'\hat{\mathrm{A}}\mathrm{A}'$ (M1) por enfoque válido

$(-6\mathbf{i} - 2\mathbf{k}) \cdot (-\mathbf{i})$

$= (\sqrt{(-6)^2 + (-2)^2})(1) \cos \mathrm{C}'\hat{\mathrm{A}}\mathrm{A}'$ (A1) por sustitución

$(-6)(-1) + (-2)(0) = \sqrt{40} \cos \mathrm{C}'\hat{\mathrm{A}}\mathrm{A}'$

$\cos \mathrm{C}'\hat{\mathrm{A}}\mathrm{A}' = \dfrac{6}{\sqrt{40}}$

$\mathrm{C}'\hat{\mathrm{A}}\mathrm{A}' = 18{,}43494882^\circ$

$\mathrm{C}'\hat{\mathrm{A}}\mathrm{A}' = 18{,}4^\circ$ A1

(ii) $\because \mathrm{C}'\mathrm{A}' = \mathrm{C}'\mathrm{A}$

$\therefore \mathrm{C}'\hat{\mathrm{A}}'\mathrm{A} = 18{,}43494882^\circ$ (A1) por enfoque correcto

$\mathrm{A}\hat{\mathrm{C}}'\mathrm{A}' + 18{,}43494882^\circ + 18{,}43494882^\circ = 180^\circ$

$\mathrm{A}\hat{\mathrm{C}}'\mathrm{A}' = 143{,}1301024^\circ$

$\mathrm{A}\hat{\mathrm{C}}'\mathrm{A}' = 143^\circ$ A1

[5]

(d) La ecuación vectorial de L:

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$$

$$\begin{cases} x = 4s \\ y = 3s \\ z = 2 + 12s \end{cases}$$ (A1) por enfoque correcto

$\therefore 4(4s) - 3(3s) + 12(2 + 12s) = -24$ (A1) por sustitución

$151s = -48 \Rightarrow s = -\dfrac{48}{151}$

$$\begin{cases} x = 4\left(-\dfrac{48}{151}\right) = -1{,}271523179 \\ y = 3\left(-\dfrac{48}{151}\right) = -0{,}9536423841 \\ z = 2 + 12\left(-\dfrac{48}{151}\right) = -1{,}814569536 \end{cases}$$ M1

Por lo tanto, las coordenadas de Q son

$(-1{,}2715; -0{,}9536; -1{,}8146)$. A1

[4]

12. (a) (i) $f'(x)=\left(\frac{1}{x^2+1}\right)(2x)$

$f'(x)=\frac{2x}{x^2+1}$ A1

$f''(x)=\frac{(x^2+1)(2)-(2x)(2x)}{(x^2+1)^2}$ (M1) por enfoque válido

$f''(x)=\frac{2x^2+2-4x^2}{(x^2+1)^2}$

$f''(x)=\frac{2-2x^2}{(x^2+1)^2}$ A1

$f^{(3)}(x)=\frac{(x^2+1)^2(-4x)-(2-2x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$ (M1) por enfoque válido

$f^{(3)}(x)=\frac{-4x^3-4x-8x+8x^3}{(x^2+1)^3}$

$f^{(3)}(x)=\frac{4x^3-12x}{(x^2+1)^3}$ A1

(ii) $f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)$

$+\frac{x^3}{3!}f^{(3)}(0)+\frac{x^4}{4!}f^{(4)}(0)+\cdots$

$f(x)=\ln(0^2+1)+x\left(\frac{2(0)}{0^2+1}\right)$

$+\frac{x^2}{2}\left(\frac{2-2(0)^2}{(0^2+1)^2}\right)+\frac{x^3}{6}\left(\frac{4(0)^3-12(0)}{(0^2+1)^3}\right)$ M2

$+\frac{x^4}{24}\left(-\frac{12(0^4-6(0)^2+1)}{(0^2+1)^4}\right)+\cdots$

$f(x)=0+x(0)+\frac{x^2}{2}(2)$

$+\frac{x^3}{6}(0)+\frac{x^4}{24}(-12)+\cdots$ A2

$f(x)=x^2-\frac{1}{2}x^4+\cdots$ A1

[10]

(b) $\operatorname{sen} x = x - \frac{x^3}{3!} + \cdots$

$\ln\left((x^2+1)^{\operatorname{sen} x}\right)$

$= \operatorname{sen} x \ln(x^2+1)$ (A1) por enfoque correcto

$= \left(x - \frac{x^3}{6} + \cdots\right)\left(x^2 - \frac{1}{2}x^4 + \cdots\right)$ M1A1

$= x^3 - \frac{1}{2}x^5 - \frac{1}{6}x^5 + \cdots$ (M1) por enfoque válido

$= x^3 - \frac{2}{3}x^5 + \cdots$ A1

[5]

(c) El valor aproximado del volumen

$= \int_{0,7}^{1,3} \pi \left(y\sqrt{\ln\left((y^2+1)^{\operatorname{sen} y}\right)} \right)^2 \mathrm{d}y$ (M1) por enfoque válido

$= \int_{0,7}^{1,3} \pi y^2 \ln\left((y^2+1)^{\operatorname{sen} y}\right) \mathrm{d}y$

$\approx \int_{0,7}^{1,3} \pi y^2 \left(y^3 - \frac{2}{3}y^5 \right) \mathrm{d}y$ A1

$\approx \int_{0,7}^{1,3} \pi \left(y^5 - \frac{2}{3}y^7 \right) \mathrm{d}y$ (M1) por enfoque válido

$\approx 0{,}3452245902$

$\approx 0{,}345$ A1

[4]

Solución de Práctica de Prueba 3 de AE NS Set 1

1. (a) (i) $\frac{2\pi}{3}$ A1

(ii) A_1

$= \pi(1)^2 - 3\left(\frac{1}{2}(1)^2 \operatorname{sen}\frac{2\pi}{3}\right)$ M1A1

$= \pi - 3\left(\frac{1}{2}\operatorname{sen}\frac{2\pi}{3}\right)$

$= \pi - \frac{3}{2}\operatorname{sen}\frac{2\pi}{3}$ A1

$= \frac{3}{2}\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right)$

$= \left(\frac{1}{2}+1\right)\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right)$ AG

[4]

(b) (i) $\frac{\pi}{3}$ A1

(ii) $\frac{1}{2}\operatorname{sen}\frac{\pi}{3}$ A1

(iii) A_2

$= \pi(1)^2 - \frac{1}{2}\operatorname{sen}\frac{2\pi}{3} - 4\left(\frac{1}{2}\operatorname{sen}\frac{\pi}{3}\right)$ M1A1

$= \pi - \frac{1}{2}\operatorname{sen}\frac{2\pi}{3} - 2\operatorname{sen}\frac{\pi}{3}$

$= \frac{\pi}{3} - \frac{1}{2}\operatorname{sen}\frac{2\pi}{3} + \frac{2\pi}{3} - 2\operatorname{sen}\frac{\pi}{3}$ M1

$= \frac{1}{2}\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right) + 2\left(\frac{\pi}{3} - \operatorname{sen}\frac{\pi}{3}\right)$ AG

[5]

(c) (i) $Q_2\hat{O}Q$

$= \frac{2\pi}{3} \div 3$ (M1) por enfoque válido

$= \frac{2\pi}{9}$ A1

(ii) A_3

$= \pi(1)^2 - \frac{1}{2}\text{sen}\frac{2\pi}{3} - 6\left(\frac{1}{2}\text{sen}\frac{2\pi}{9}\right)$ M1A1

$= \pi - \frac{1}{2}\text{sen}\frac{2\pi}{3} - 3\,\text{sen}\frac{2\pi}{9}$

$= \frac{\pi}{3} - \frac{1}{2}\text{sen}\frac{2\pi}{3} + \frac{2\pi}{3} - 3\,\text{sen}\frac{2\pi}{9}$ M1

$= \frac{1}{2}\left(\frac{2\pi}{3} - \text{sen}\frac{2\pi}{3}\right) + 3\left(\frac{2\pi}{9} - \text{sen}\frac{2\pi}{9}\right)$ A1

[6]

(d) (i) A_n

$= \pi(1)^2 - \frac{1}{2}\text{sen}\frac{2\pi}{3} - 2n\left(\frac{1}{2}\text{sen}\left(\frac{2\pi}{3} \div n\right)\right)$ M1A1

$= \pi - \frac{1}{2}\text{sen}\frac{2\pi}{3} - n\,\text{sen}\frac{2\pi}{3n}$

$= \frac{\pi}{3} - \frac{1}{2}\text{sen}\frac{2\pi}{3} + \frac{2\pi}{3} - n\,\text{sen}\frac{2\pi}{3n}$ M1

$= \frac{1}{2}\left(\frac{2\pi}{3} - \text{sen}\frac{2\pi}{3}\right) + n\left(\frac{2\pi}{3n} - \text{sen}\frac{2\pi}{3n}\right)$ A1

$\therefore f(n) = \frac{2\pi}{3n} - \text{sen}\frac{2\pi}{3n}$ A1

(ii) $f(n)$ representa el doble de área del segmento del sector POQ_1. A1

[6]

(e) $\lim_{n\to\infty} f(n)$

$$= \lim_{n\to\infty}\left(\frac{2\pi}{3n} - \operatorname{sen}\frac{2\pi}{3n}\right)$$

$$= \frac{2\pi}{3}\lim_{n\to\infty}\frac{1}{n} - \lim_{n\to\infty}\operatorname{sen}\frac{2\pi}{3n}$$ M1

$$= \frac{2\pi}{3}\lim_{n\to\infty}\frac{1}{n} - \operatorname{sen}\left(\frac{2\pi}{3}\lim_{n\to\infty}\frac{1}{n}\right)$$

$$= \frac{2\pi}{3}(0) - \operatorname{sen}\left(\frac{2\pi}{3}(0)\right)$$

$$= 0$$ A1

[2]

(f) (i) $\frac{3}{2}\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right)$ A2

(ii) El mayor valor posible de v

$$= \lim_{n\to\infty} A_n$$ M1

$$= \lim_{n\to\infty}\left(\frac{1}{2}\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right) + n\cdot f(n)\right)$$

$$= \frac{1}{2}\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right)$$ A1

[4]

2. (a) (i) $w^2 - w + 1 = 0$

$$w = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$ (A1) por sustitución

$$w = \frac{1 \pm \sqrt{-3}}{2}$$

$$w = \frac{1 \pm \sqrt{3}\mathrm{i}}{2}$$

$$w = \frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$$

$$w = \cos\frac{\pi}{3} + \mathrm{i}\,\mathrm{sen}\,\frac{\pi}{3}$$ o

$$w = \cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{3}\right)$$ A2

(ii) $u^4 - u^2 + 1 = 0$

$(u^2)^2 - u^2 + 1 = 0$ M1

$u^2 = \cos\frac{\pi}{3} + \mathrm{i}\,\mathrm{sen}\frac{\pi}{3}$ o

$u^2 = \cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{3}\right)$

$u = \cos\left(\frac{\frac{\pi}{3} + 2\pi k}{2}\right) + \mathrm{i}\,\mathrm{sen}\left(\frac{\frac{\pi}{3} + 2\pi k}{2}\right)$ o

$u = \cos\left(\frac{-\frac{\pi}{3} + 2\pi k}{2}\right) + \mathrm{i}\,\mathrm{sen}\left(\frac{-\frac{\pi}{3} + 2\pi k}{2}\right)$

$(k = 0, 1)$ A1

$u = \cos\frac{\pi}{6} + \mathrm{i}\,\mathrm{sen}\frac{\pi}{6}$, $u = \cos\frac{7\pi}{6} + \mathrm{i}\,\mathrm{sen}\frac{7\pi}{6}$,

$u = \cos\left(-\frac{\pi}{6}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{6}\right)$ o

$u = \cos\frac{5\pi}{6} + \mathrm{i}\,\mathrm{sen}\frac{5\pi}{6}$ A2

Por lo tanto, las raíces requeridas son

$\cos\left(-\frac{5\pi}{6}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{5\pi}{6}\right)$,

$\cos\left(-\frac{\pi}{6}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{6}\right)$, $\cos\frac{\pi}{6} + \mathrm{i}\,\mathrm{sen}\frac{\pi}{6}$ y

$\cos\frac{5\pi}{6} + \mathrm{i}\,\mathrm{sen}\frac{5\pi}{6}$. AG

(iii) $z^{2n} - z^n + 1 = 0$

$(z^n)^2 - z^n + 1 = 0$

$z^n = \cos\frac{\pi}{3} + \mathrm{i}\,\mathrm{sen}\frac{\pi}{3}$ o

$z^n = \cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{3}\right)$

$z = \cos\left(\frac{\frac{\pi}{3} + 2\pi k}{n}\right) + \mathrm{i}\,\mathrm{sen}\left(\frac{\frac{\pi}{3} + 2\pi k}{n}\right)$ o

$z = \cos\left(\frac{-\frac{\pi}{3} + 2\pi k}{n}\right) + \mathrm{i}\,\mathrm{sen}\left(\frac{-\frac{\pi}{3} + 2\pi k}{n}\right)$

$(k = 0, 1, 2, \cdots, n-1)$ A1

$z = \cos\left(\frac{\pi}{3n} + \frac{2\pi}{n}k\right) + \mathrm{i}\,\mathrm{sen}\left(\frac{\pi}{3n} + \frac{2\pi}{n}k\right)$ o

$z = \cos\left(-\frac{\pi}{3n} + \frac{2\pi}{n}k\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{3n} + \frac{2\pi}{n}k\right)$

$(k = 0, 1, 2, \cdots, n-1)$

$z = \cos\frac{\pi + 6\pi k}{3n} + \mathrm{i}\,\mathrm{sen}\frac{\pi + 6\pi k}{3n}$ o

$z = \cos\frac{-\pi + 6\pi k}{3n} + \mathrm{i}\,\mathrm{sen}\frac{-\pi + 6\pi k}{3n}$

$(k = 0, 1, 2, \cdots, n-1)$ A1

Por lo tanto, las raíces requeridas son

$\cos\left(-\frac{\pi}{3n}\right) + \mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{3n}\right)$, $\cos\frac{\pi}{3n} + \mathrm{i}\,\mathrm{sen}\frac{\pi}{3n}$,

$\cos\frac{5\pi}{3n} + \mathrm{i}\,\mathrm{sen}\frac{5\pi}{3n}$, $\cos\frac{7\pi}{3n} + \mathrm{i}\,\mathrm{sen}\frac{7\pi}{3n}$, $\cdots$,

$\cos\frac{(6n-7)\pi}{3n} + \mathrm{i}\,\mathrm{sen}\frac{(6n-7)\pi}{3n}$ y

$\cos\frac{(6n-5)\pi}{3n} + \mathrm{i}\,\mathrm{sen}\frac{(6n-5)\pi}{3n}$. A3

[12]

(b) (i) $(z-(\cos\theta+\mathrm{i}\,\mathrm{sen}\,\theta))(z-(\cos(-\theta)+\mathrm{i}\,\mathrm{sen}\,(-\theta)))$

$=(z-\cos\theta-\mathrm{i}\,\mathrm{sen}\,\theta)(z-\cos\theta+\mathrm{i}\,\mathrm{sen}\,\theta)$

$=z^2-z\cos\theta+\mathrm{i}z\,\mathrm{sen}\,\theta-z\cos\theta+\cos^2\theta$

$-\mathrm{i}\,\mathrm{sen}\,\theta\cos\theta-\mathrm{i}z\,\mathrm{sen}\,\theta+\mathrm{i}\,\mathrm{sen}\,\theta\cos\theta+\mathrm{sen}^2\theta$ M1

$=z^2-2z\cos\theta+1$ A1

(ii) u^4-u^2+1

$=\left(u-\left(\cos\frac{\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{\pi}{6}\right)\right)$

$\left(u-\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\,\mathrm{sen}\left(-\frac{\pi}{6}\right)\right)\right)$

$\left(u-\left(\cos\frac{5\pi}{6}+\mathrm{i}\,\mathrm{sen}\frac{5\pi}{6}\right)\right)$ M1A1

$\left(u-\left(\cos\left(-\frac{5\pi}{6}\right)+\mathrm{i}\,\mathrm{sen}\left(-\frac{5\pi}{6}\right)\right)\right)$

$=\left(u^2-2u\cos\frac{\pi}{6}+1\right)\left(u^2-2u\cos\frac{5\pi}{6}+1\right)$ AG

(iii) Las raíces de la ecuación $z^6-z^3+1=0$

son $\mathrm{cis}\frac{\pi}{9}$, $\mathrm{cis}\left(-\frac{\pi}{9}\right)$, $\mathrm{cis}\frac{5\pi}{9}$, $\mathrm{cis}\left(-\frac{5\pi}{9}\right)$,

$\mathrm{cis}\frac{7\pi}{9}$ y $\mathrm{cis}\left(-\frac{7\pi}{9}\right)$. (A1) por valores correctos

z^6-z^3+1

$=\left(z-\mathrm{cis}\frac{\pi}{9}\right)\left(z-\mathrm{cis}\left(-\frac{\pi}{9}\right)\right)\left(z-\mathrm{cis}\frac{5\pi}{9}\right)$

$\left(z-\mathrm{cis}\left(-\frac{5\pi}{9}\right)\right)\left(z-\mathrm{cis}\frac{7\pi}{9}\right)$ A1

$\left(z-\mathrm{cis}\left(-\frac{7\pi}{9}\right)\right)$

$=\left(z^2-2z\cos\frac{\pi}{9}+1\right)\left(z^2-2z\cos\frac{5\pi}{9}+1\right)$ A1

$\left(z^2-2z\cos\frac{7\pi}{9}+1\right)$

(iv) $z^{2n} - z^n + 1 = 0$

$$= \left(z^2 - 2z\cos\frac{\pi}{3n} + 1\right)\left(z^2 - 2z\cos\frac{5\pi}{3n} + 1\right)$$

$$\left(z^2 - 2z\cos\frac{7\pi}{3n} + 1\right)\cdots$$

$$\left(z^2 - 2z\cos\left(\pi - \frac{5\pi}{3n}\right) + 1\right)$$ A2

$$\left(z^2 - 2z\cos\left(\pi - \frac{\pi}{3n}\right) + 1\right)$$

[9]

(c) $u^4 - u^2 + 1 = \left(u^2 - 2u\cos\frac{\pi}{6} + 1\right)\left(u^2 - 2u\cos\frac{5\pi}{6} + 1\right)$

Cuando $u = \mathrm{i}$,

$\mathrm{i}^4 - \mathrm{i}^2 + 1 = \left(\mathrm{i}^2 - 2\mathrm{i}\cos\frac{\pi}{6} + 1\right)\left(\mathrm{i}^2 - 2\mathrm{i}\cos\frac{5\pi}{6} + 1\right)$ M1

$1 - (-1) + 1 = \left(-1 - 2\mathrm{i}\cos\frac{\pi}{6} + 1\right)\left(-1 - 2\mathrm{i}\cos\frac{5\pi}{6} + 1\right)$ A1

$3 = \left(-2\mathrm{i}\cos\frac{\pi}{6}\right)\left(-2\mathrm{i}\cos\frac{5\pi}{6}\right)$

$3 = 4\mathrm{i}^2\cos\frac{\pi}{6}\cos\frac{5\pi}{6}$ A1

$3 = -4\cos\frac{\pi}{6}\cos\frac{5\pi}{6}$

$\cos\frac{\pi}{6}\cos\frac{5\pi}{6} = -\frac{3}{4}$ AG

[3]

(d)

$$z^6 - z^3 + 1 = \left(z^2 - 2z\cos\frac{\pi}{9} + 1\right)\left(z^2 - 2z\cos\frac{5\pi}{9} + 1\right)$$
$$\left(z^2 - 2z\cos\frac{7\pi}{9} + 1\right)$$

Cuando $z = \mathrm{i}$,

$$\mathrm{i}^6 - \mathrm{i}^3 + 1 = \left(\mathrm{i}^2 - 2\mathrm{i}\cos\frac{\pi}{9} + 1\right)\left(\mathrm{i}^2 - 2\mathrm{i}\cos\frac{5\pi}{9} + 1\right)$$
$$\left(\mathrm{i}^2 - 2\mathrm{i}\cos\frac{7\pi}{9} + 1\right)$$

(M1) por enfoque válido

$$-1 - (-\mathrm{i}) + 1 = \left(-1 - 2\mathrm{i}\cos\frac{\pi}{9} + 1\right)$$
$$\left(-1 - 2\mathrm{i}\cos\frac{5\pi}{9} + 1\right)\left(-1 - 2\mathrm{i}\cos\frac{7\pi}{9} + 1\right)$$

A1

$$\mathrm{i} = \left(-2\mathrm{i}\cos\frac{\pi}{9}\right)\left(-2\mathrm{i}\cos\frac{5\pi}{9}\right)\left(-2\mathrm{i}\cos\frac{7\pi}{9}\right)$$

$$\mathrm{i} = -8\mathrm{i}^3\cos\frac{\pi}{9}\cos\frac{5\pi}{9}\cos\frac{7\pi}{9}$$

A1

$$\mathrm{i} = 8\mathrm{i}\cos\frac{\pi}{9}\cos\frac{5\pi}{9}\cos\frac{7\pi}{9}$$

$$\cos\frac{\pi}{9}\cos\frac{5\pi}{9}\cos\frac{7\pi}{9} = \frac{1}{8}$$

A1

[4]

Solución de Práctica de Prueba 1 de AE NS Set 2

Sección A

1. (a) (i) 7 A1

(ii) 1 A1

[2]

(b) $(f \circ g)(x)$

$= (g(x))^2$ (A1) por sustitución

$= (3-4x)^2$

$= 9-24x+16x^2$ A1

[2]

(c) $y = 3-4x$

$\Rightarrow x = 3-4y$ (A1) por enfoque correcto

$4y = 3-x$

$y = \dfrac{3-x}{4}$

$\therefore g^{-1}(x) = \dfrac{3-x}{4}$ A1

[2]

2. (a) Lado derecho

$$=\frac{1\times 49}{1\times 49}+\frac{2\times 7}{7\times 7}+\frac{5}{49}$$ M1

$$=\frac{49+14+5}{49}$$ A1

$$=\frac{68}{49}=\text{Lado izquierdo}$$

$$\therefore \frac{68}{49}=1+\frac{2}{7}+\frac{5}{49}$$ AG

[2]

(b) Lado derecho

$$=\frac{1\times (m+2)^2}{1\times (m+2)^2}+\frac{2\times (m+2)}{(m+2)\times (m+2)}+\frac{5}{(m+2)^2}$$ M1

$$=\frac{(m^2+4m+4)+(2m+4)+5}{(m+2)^2}$$ M1A1

$$=\frac{m^2+6m+9+4}{(m+2)^2}$$

$$=\frac{(m+3)^2+4}{(m+2)^2}=\text{Lado izquierdo}$$

$$\therefore \frac{(m+3)^2+4}{(m+2)^2}\equiv 1+\frac{2}{m+2}+\frac{5}{(m+2)^2}$$ para $m\neq -2$ AG

[3]

3. $P(2)=0$

$a(2)^3+b(2)^2-10(2)+24=0$ (M1) por teorema del factor

$4b=-4-8a$

$b=-1-2a$ A1

$P(-3)=0$

$a(-3)^3+b(-3)^2-10(-3)+24=0$

$-27a+9b+30+24=0$

$\therefore -27a+9(-1-2a)+30+24=0$ (M1) por sustitución

$-27a-9-18a+30+24=0$

$-45a=-45$

$a=1$ A1

$b=-1-2(1)$

$b=-3$ A1

[5]

4. (a) El discriminante de $f(x)$

$= b^2 - 4ac$

$= (8-p)^2 - 4\left(1+2p-\frac{3}{8}p^2\right)(-2)$ M1A1

$= 64 - 16p + p^2 + 8 + 16p - 3p^2$ A1

$= 72 - 2p^2$ AG

[3]

(b) $f(x) = 0$ tiene dos raíces iguales

$\therefore 72 - 2p^2 = 0$ (M1) por ecuación

$2p^2 = 72$

$p^2 = 36$

$p = -6$ o $p = 6$ A2

[3]

(c) $p = 6$

$\therefore \left(1+2(6)-\frac{3}{8}(6)^2\right)x^2 + (8-6)x - 2 = 0$ (M1) por ecuación

$-\frac{1}{2}x^2 + 2x - 2 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$ A1

[2]

5. $9\log_{27}(x+1) = 1 + \log_3(3+x+x^2)$

$\frac{9\log_3(x+1)}{\log_3 27} = \log_3 3 + \log_3(3+x+x^2)$ (M1)(A1) por cambio de base

$\frac{9\log_3(x+1)}{3} = \log_3 3(3+x+x^2)$ (A1) por enfoque correcto

$3\log_3(x+1) = \log_3 3(3+x+x^2)$

$\log_3(x+1)^3 = \log_3 3(3+x+x^2)$ A1

$\therefore (x+1)^3 = 3(3+x+x^2)$ M1

$x^3 + 3x^2 + 3x + 1 = 9 + 3x + 3x^2$

$x^3 = 8$ A1

$x = \sqrt[3]{8}$

$x = 2$ A1

[7]

6. (a) $r = \dfrac{20\cos^4 \alpha}{30\cos^2 \alpha}$ (M1) por enfoque válido

$r = \dfrac{2}{3}\cos^2 \alpha$ A1

[2]

(b) $\pi \le \alpha \le \dfrac{4}{3}\pi$

$\therefore \cos\pi \le \cos\alpha \le \cos\dfrac{4}{3}\pi$ (M1) por enfoque válido

$-1 \le \cos\alpha \le -\dfrac{1}{2}$

$\dfrac{1}{4} \le \cos^2 \alpha \le 1$

$\dfrac{1}{6} \le \dfrac{2}{3}\cos^2 \alpha \le \dfrac{2}{3}$

$\therefore \dfrac{1}{6} \le r \le \dfrac{2}{3}$ A1

[2]

(c) $S_\infty = \dfrac{30\cos^2 \alpha}{1 - \dfrac{2}{3}\cos^2 \alpha}$ A1

$S_\infty = \dfrac{30\cos^2 \alpha}{\text{sen}^2\alpha + \cos^2 \alpha - \dfrac{2}{3}\cos^2 \alpha}$ M1

$S_\infty = \dfrac{30\cos^2 \alpha}{\text{sen}^2\alpha + \dfrac{1}{3}\cos^2 \alpha}$ A1

$S_\infty = \dfrac{30}{\tan^2 \alpha + \dfrac{1}{3}}$ A1

$S_\infty = \dfrac{90}{3\tan^2 \alpha + 1}$ AG

[4]

7. Cuando $n=1$,

Lado izquierdo $=1^2$

Lado izquierdo $=1$

Lado derecho $=\frac{4}{3}(1)^3$

Lado derecho $=\frac{4}{3}$

Por lo tanto, el enunciado es verdadero cuando $n=1$. R1

Asumir que el enunciado es verdadero cuando $n=k$. M1

$1^2+2^2+\ldots+k^2\leq\frac{4}{3}k^3$

Cuando $n=k+1$,

$1^2+2^2+\ldots+k^2+(k+1)^2$

$\leq\frac{4}{3}k^3+(k+1)^2$ M1A1

$=\frac{4k^3+3(k^2+2k+1)}{3}$ A1

$=\frac{4k^3+3k^2+6k+3}{3}$

$\leq\frac{4k^3+12k^2+12k+4}{3}$ A1

$=\frac{4(k^3+3k^2+3k+1)}{3}$

$=\frac{4}{3}(k+1)^3$

Por lo tanto, el enunciado es verdadero cuando $n=k+1$.

A partir de lo anterior, el enunciado es verdadero para todo $n\in\mathbb{Z}^+$. R1

[7]

8. $\lim_{x\to 0}\frac{1-e^{x^2}}{1-\sec x}$

$=\lim_{x\to 0}\frac{0-(e^{x^2})(2x)}{-\sec x\tan x}\left(\because \frac{0}{0}\right)$ M1A2

$=\lim_{x\to 0}\frac{2xe^{x^2}}{\sec x\tan x}$

$=\lim_{x\to 0}\frac{(2)(e^{x^2})+(2x)(e^{x^2})(2x)}{(\sec x\tan x)(\tan x)+(\sec x)(\sec^2 x)}\left(\because \frac{0}{0}\right)$ A2

$=\lim_{x\to 0}\frac{2e^{x^2}+4x^2e^{x^2}}{\sec x\tan^2 x+\sec^3 x}$

$=\frac{2e^0+4(0)^2e^0}{\sec 0\tan^2 0+\sec^3 0}$ M1

$=\frac{2+0}{0+1}$

$=2$ A1

[7]

9. (a) $-\pi a$ A1

[1]

(b) $\int_{-\pi a}^{a}|x|\,\mathrm{d}x=1$ A1

$\int_{-\pi a}^{0}-x\mathrm{d}x+\int_{0}^{a}x\mathrm{d}x=1$

$\left[-\frac{1}{2}x^2\right]_{-\pi a}^{0}+\left[\frac{1}{2}x^2\right]_{0}^{a}=1$ A1

$\left(0-\left(-\frac{1}{2}\pi^2a^2\right)\right)+\left(\frac{1}{2}a^2-0\right)=1$

$\frac{1}{2}\pi^2a^2+\frac{1}{2}a^2=1$ M1

$a^2(\pi^2+1)=2$

$a^2=\frac{2}{\pi^2+1}$ A1

$a=-\sqrt{\frac{2}{\pi^2+1}}$ (*Rechazado*) o $a=\sqrt{\frac{2}{\pi^2+1}}$

Por tanto, $a=\sqrt{\frac{2}{\pi^2+1}}$. AG

[4]

Sección B

10.	(a)	$2r+h=20$	(A1) por enfoque correcto
		$2r=20-h$	
		$r=10-\frac{1}{2}h$	A1
			[2]
	(b)	$V=\pi r^2 h$	
		$V=\pi\left(10-\frac{1}{2}h\right)^2 h$	(A1) por sustitución
		$V=100\pi h-10\pi h^2+\frac{1}{4}\pi h^3$	A1
			[2]
	(c)	$Q=(3)(2\pi rh)+(4)(\pi r^2)$	M1A1
		$Q=6\pi\left(10-\frac{1}{2}h\right)h+4\pi\left(10-\frac{1}{2}h\right)^2$	M1
		$Q=60\pi h-3\pi h^2+400\pi-40\pi h+\pi h^2$	A1
		$Q=400\pi+20\pi h-2\pi h^2$	
		$Q=2\pi(200+10h-h^2)$	AG
			[4]
	(d)	$\frac{dQ}{dh}=2\pi(0+10(1)-2h)$	(A1) por derivadas correctas
		$\frac{dQ}{dh}=4\pi(5-h)$	A1
		$\frac{dQ}{dh}=0$	(M1) por ecuación
		$\therefore 4\pi(5-h)=0$	A1
		$h=5$	A1
		El valor máximo de Q	
		$=2\pi(200+10(5)-(5)^2)$	(M1) por sustitución
		$=450\pi$	A1
			[7]

11. (a) $\overrightarrow{BD} = \begin{pmatrix} -9 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 9 \\ -9 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -9 \\ -9 \\ 9 \end{pmatrix}$ A1

La ecuación vectorial de BD:

$\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -9 \end{pmatrix} + t\begin{pmatrix} -9 \\ -9 \\ 9 \end{pmatrix}$

$\begin{cases} x = -9t \\ y = 9 - 9t \\ z = -9 + 9t \end{cases}$ A1

$\overrightarrow{CE} = \begin{pmatrix} -9t \\ 9-9t \\ -9+9t \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ -9 \end{pmatrix}$

$\overrightarrow{CE} = \begin{pmatrix} -9t \\ 9-9t \\ 9t \end{pmatrix}$ A1

$\overrightarrow{CE} \cdot \overrightarrow{BD} = 0$

$\therefore (-9t)(-9) + (9-9t)(-9) + (9t)(9) = 0$ M1

$81t - 81 + 81t + 81t = 0$

$243t = 81$

$t = \frac{1}{3}$ A1

$\therefore \begin{cases} x = -9\left(\frac{1}{3}\right) = -3 \\ y = 9 - 9\left(\frac{1}{3}\right) = 6 \\ z = -9 + 9\left(\frac{1}{3}\right) = -6 \end{cases}$ M1

Por tanto, las coordenadas de E son

$(-3, 6, -6)$. AG

[6]

(b) $\vec{BA} = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 9 \\ -9 \end{pmatrix}$

$\vec{BA} = \begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix}$ (A1) por valores correctos

$\vec{BC} = \begin{pmatrix} 0 \\ 0 \\ -9 \end{pmatrix} - \begin{pmatrix} 0 \\ 9 \\ -9 \end{pmatrix}$

$\vec{BC} = \begin{pmatrix} 0 \\ -9 \\ 0 \end{pmatrix}$ (A1) por valores correctos

$\mathbf{n}_1 = \vec{BA} \times \vec{BD}$ (M1) por enfoque válido

$\mathbf{n}_1 = \begin{pmatrix} 0 \\ 0 \\ 9 \end{pmatrix} \times \begin{pmatrix} -9 \\ -9 \\ 9 \end{pmatrix}$

$\mathbf{n}_1 = \begin{pmatrix} (0)(9) - (9)(-9) \\ (9)(-9) - (0)(9) \\ (0)(-9) - (0)(-9) \end{pmatrix}$

$\mathbf{n}_1 = \begin{pmatrix} 81 \\ -81 \\ 0 \end{pmatrix}$ A1

$\mathbf{n}_2 = \vec{BC} \times \vec{BD}$ (M1) por enfoque válido

$\mathbf{n}_2 = \begin{pmatrix} 0 \\ -9 \\ 0 \end{pmatrix} \times \begin{pmatrix} -9 \\ -9 \\ 9 \end{pmatrix}$

$\mathbf{n}_2 = \begin{pmatrix} (-9)(9) - (0)(-9) \\ (0)(-9) - (0)(9) \\ (0)(-9) - (-9)(-9) \end{pmatrix}$

$\mathbf{n}_2 = \begin{pmatrix} -81 \\ 0 \\ -81 \end{pmatrix}$ A1

$\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2|\cos\theta$ (M1) por enfoque válido

$(81)(-81)+(-81)(0)+(0)(-81)$
$= (\sqrt{81^2+(-81)^2})(\sqrt{(-81)^2+(-81)^2})\cos\theta$ A1

$-81^2 = 2(81)^2\cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$ A1

[9]

(c) El área de OABC

$= (\text{OA})(\text{OC})$

$= (9)(9)$

$= 81$ (A1) por valor correcto

$\therefore \frac{1}{3}(81)(\text{OD}) + \frac{1}{3}(81)(\text{OF}) = 783$ (M1) por ecuación

$\frac{1}{3}(81)(9) + \frac{1}{3}(81)(\text{OF}) = 783$

$27\text{OF} = 540$

$\text{OF} = 20$ A1

$\therefore \text{DF} = 9 + 20$

$\text{DF} = 29$ A1

[4]

12. (a) $\frac{\mathrm{d}a}{\mathrm{d}t} - 2a^2 = 50$

$\frac{\mathrm{d}a}{\mathrm{d}t} = 2a^2 + 50$

$\frac{\mathrm{d}a}{\mathrm{d}t} = 2(a^2 + 25)$

$\frac{1}{a^2+25}\mathrm{d}a = 2\mathrm{d}t$ (M1) por enfoque válido

$\int \frac{1}{a^2+25}\mathrm{d}a = \int 2\mathrm{d}t$ (A1) por enfoque correcto

$\int \frac{1}{a^2+5^2}\mathrm{d}a = \int 2\mathrm{d}t$

$\frac{1}{5}\arctan\frac{a}{5} = 2t + C$ A1

$\arctan\frac{a}{5} = 10t + C$

$\frac{a}{5} = \tan(10t + C)$

$a = 5\tan(10t + C)$ A1

$5 = 5\tan(10(0) + C)$ (M1) por sustitución

$1 = \tan C$

$\tan C = \tan\frac{\pi}{4}$

$C = \frac{\pi}{4}$ (A1) por valor correcto

$\therefore a = 5\tan\left(10t + \frac{\pi}{4}\right)$ A1

[7]

(b) $\frac{\mathrm{d}v}{\mathrm{d}t} = 5\tan\left(10t + \frac{\pi}{4}\right)$

$\mathrm{d}v = \frac{5\sin\left(10t + \frac{\pi}{4}\right)}{\cos\left(10t + \frac{\pi}{4}\right)}\mathrm{d}t$

$\int \mathrm{d}v = \int \frac{5\sin\left(10t + \frac{\pi}{4}\right)}{\cos\left(10t + \frac{\pi}{4}\right)}\mathrm{d}t$ (A1) por enfoque correcto

Sea $u = \cos\left(10t + \frac{\pi}{4}\right)$. (M1) por sustitución

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -10\,\mathrm{sen}\left(10t + \frac{\pi}{4}\right) \Rightarrow 5\,\mathrm{sen}\left(10t + \frac{\pi}{4}\right)\mathrm{d}t = -\frac{1}{2}\mathrm{d}u$$

$$\therefore \int \mathrm{d}v = -\frac{1}{2}\int \frac{1}{u}\mathrm{d}u$$ (A1) por enfoque correcto

$$v = -\frac{1}{2}\ln u + D$$

$$v = -\frac{1}{2}\ln\left|\cos\left(10t + \frac{\pi}{4}\right)\right| + D$$ A1

$$\ln 2^{\frac{1}{4}} = -\frac{1}{2}\ln\left|\cos\left(10(0) + \frac{\pi}{4}\right)\right| + D$$ (M1) por sustitución

$$\frac{1}{4}\ln 2 = -\frac{1}{2}\ln\frac{\sqrt{2}}{2} + D$$

$$\frac{1}{4}\ln 2 = \frac{1}{4}\ln 2 + D$$

$$D = 0$$ (A1) por valor correcto

$$\therefore v = -\frac{1}{2}\ln\left|\cos\left(10t + \frac{\pi}{4}\right)\right|$$ A1

[7]

(c) $$v = -\frac{1}{2}\ln\left|\cos\left(10t + \frac{\pi}{4}\right)\right|$$

$$v = \frac{1}{2}\ln\left|\frac{1}{\cos\left(10t + \frac{\pi}{4}\right)}\right|$$ M1

$$v = \frac{1}{2}\ln\left|\sec\left(10t + \frac{\pi}{4}\right)\right|$$ A1

$$v = \frac{1}{4}\ln\left(\sec^2\left(10t + \frac{\pi}{4}\right)\right)$$

$$v = \frac{1}{4}\ln\left(\tan^2\left(10t + \frac{\pi}{4}\right) + 1\right)$$ A1

$$\therefore v = \frac{1}{4}\ln\left(\left(\frac{a}{5}\right)^2 + 1\right)$$ A1

$$v = \frac{1}{4}\ln\left(\frac{a^2}{25} + 1\right)$$

$$v = \frac{1}{4}\ln\left(\frac{a^2 + 25}{25}\right)$$ AG

[4]

Solución de Práctica de Prueba 2 de AE NS Set 2

Sección A

1.

$$\left(kx-\frac{4}{x}\right)^8=(kx)^8+\binom{8}{1}(kx)^7\left(-\frac{4}{x}\right)+\binom{8}{2}(kx)^6\left(-\frac{4}{x}\right)^2$$

$$+\binom{8}{3}(kx)^5\left(-\frac{4}{x}\right)^3+\binom{8}{4}(kx)^4\left(-\frac{4}{x}\right)^4+\ldots$$

(M1)(A1) por enfoque correcto

$$\left(kx-\frac{4}{x}\right)^8=k^8x^8+8k^7x^7\left(-\frac{4}{x}\right)+28k^6x^6\left(\frac{16}{x^2}\right)$$

$$+56k^5x^5\left(-\frac{64}{x^3}\right)+70k^4x^4\left(\frac{256}{x^4}\right)+\ldots$$

(A1) por simplificación

$$\left(kx-\frac{4}{x}\right)^8=k^8x^8-32k^7x^6+448k^6x^4$$

$$-3584k^5x^2+17920k^4+\ldots$$

A1

$\therefore 448k^6:17920k^4=9:40$ A1

$$\frac{448k^6}{17920k^4}=\frac{9}{40}$$

$$\frac{k^2}{40}=\frac{9}{40}$$

$k=-3$ o $k=3$ (*Rechazada*) A1

[6]

2. (a) $A = 2\pi r^2 + 2\pi rh + 2\pi r^2$ (M2) por ecuación

$135\pi = 4\pi r^2 + 2\pi r(3,5)$ (A1) por sustitución

$135 = 4r^2 + 7r$

$4r^2 + 7r - 135 = 0$ (M1) por ecuación cuadrática

$(4r + 27)(r - 5) = 0$

$4r + 27 = 0$ o $r - 5 = 0$

$r = -\frac{27}{4}$ (*Rechazada*) o $r = 5$ mm A1

[5]

(b) El volumen

$= \frac{4}{3}\pi r^3 + \pi r^2 h$ (M1) por enfoque válido

$= \frac{4}{3}\pi(5)^3 + \pi(5)^2(3,5)$

$= 798,4881328 \text{ mm}^3$

$= 798 \text{ mm}^3$ A1

[2]

3. (a) (i) $\cos \text{A}\hat{\text{B}}\text{C} = \frac{r^2 + (1,75r)^2 - (1,5r)^2}{2(r)(1,75r)}$ M1A1

$\cos \text{A}\hat{\text{B}}\text{C} = \frac{1,8125r^2}{3,5r^2}$ A1

$\cos \text{A}\hat{\text{B}}\text{C} = \frac{29}{56}$ AG

(ii) $\text{A}\hat{\text{B}}\text{C} = 1,026452178 \text{ rad}$

$\text{A}\hat{\text{B}}\text{C} = 1,03 \text{ rad}$ A1

[4]

(b) $\frac{1}{2}(\text{BC})^2(\text{A}\hat{\text{B}}\text{C}) = 9,89$ (M1) por ecuación

$\frac{1}{2}r^2(\pi - 1,026452178) = 9,89$ (A1) por sustitución

$r^2 = 9,35162474$

$r = 3,058042632$

$r = 3,06$ A1

[3]

4. $X \sim \mathrm{B}\left(5, \dfrac{2p}{p+2p+10}\right)$ (R1) por distribución correcta

La desviación típica de X

$= \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(1-\dfrac{2p}{3p+10}\right)}$ (A1) por sustitución

$= \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right)}$

$\therefore \sqrt{5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right)} > \dfrac{11}{10}$ (M1) por enfoque válido

$5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) > \dfrac{121}{100}$ M1

$5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) - \dfrac{121}{100} > 0$ A1

Considerando el gráfico de

$y = 5\left(\dfrac{2p}{3p+10}\right)\left(\dfrac{p+10}{3p+10}\right) - \dfrac{121}{100}$,

$5,3435147 < p < 25,443002$.

Por lo tanto, el mayor valor de p es 25. A1

[6]

5. $v = \int (8-8t)\mathrm{d}t$ (M1) por integral indefinida

$v = 8t - 8\left(\dfrac{1}{2}t^2\right) + C$ A1

$v = 8t - 4t^2 + C$

La velocidad inicial

$= 8(0) - 4(0)^2 + C$ (M1) por enfoque válido

$= C$

La diferencia entre las velocidades es de $4\ \mathrm{ms}^{-1}$

$\therefore 8t - 4t^2 + C = C + 4$ o $\therefore 8t - 4t^2 + C = C - 4$ (A1) por enfoque correcto

$4t^2 - 8t + 4 = 0$ o $4t^2 - 8t - 4 = 0$ A2

$4(t-1)^2 = 0$ o $t = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-4)}}{2(4)}$

$t = 1$ o $t = 2,414213562$,

$t = -0,4142135624$ (*Rechazada*)

$\therefore m = 1$ o $m = 2,41$ A2

[8]

6. (a) Por utilizar operaciones con las filas, el sistema

$$\left(\begin{array}{ccc|c} 2 & -1 & -3 & 3 \\ 1 & -4 & -6 & -17 \\ 3 & 1 & 2 & 21 \end{array}\right)$$ se reduce a

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right).$$ (M1) por enfoque válido

Por lo tanto, las coordenadas de P son $(5, 4, 1)$. A3

[4]

(b) $\mathbf{r} = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -4 \\ -6 \end{pmatrix}$ A2

[2]

7. (a) $\{x : -5 \le x \le 1\}$ A2

[2]

(b) $f(x) = 2 - (x-1)^2$

$y = 2 - (x-1)^2$

$\Rightarrow x = 2 - (y-1)^2$ (M1) for intercambiar variables

$(y-1)^2 = 2 - x$

$y - 1 = \sqrt{2-x}$ (*Rechazada*) o $y - 1 = -\sqrt{2-x}$ A1

$y = -\sqrt{2-x} + 1$

$\therefore f^{-1}(x) = -\sqrt{2-x} + 1$ A1

[3]

(c) $(g^{-1} \circ f^{-1})(x) = \dfrac{x}{3}$

$f^{-1}(x) = g\left(\dfrac{x}{3}\right)$ M1

$g\left(\dfrac{x}{3}\right) = -\sqrt{2-x} + 1$

$g\left(3\left(\dfrac{x}{3}\right)\right) = -\sqrt{2-3x} + 1$ A1

$\therefore g(x) = -\sqrt{2-3x} + 1$ A1

[3]

8. $\dfrac{2\pi}{B} = 2(4-0)$

$\dfrac{2\pi}{B} = 8$

$B = \dfrac{\pi}{4}$ A1

$5+\pi = A\sec\dfrac{\pi}{4}(0) + C$

$5+\pi = A + C$

$C = 5+\pi - A$

$5-\pi = A\sec\dfrac{\pi}{4}(4) + C$

$\therefore 5-\pi = A(-1)+5+\pi - A$ (M1) por sustitución

$-2\pi = -2A$

$A = \pi$ A1

$C = 5+\pi-\pi$

$C = 5$ A1

[4]

9. (a) El número total de maneras posibles es

$= \dfrac{14!}{14\times 2}$ (A2) por fórmula correcta

$= 3113510400$ A1

[3]

(b) El número de maneras posibles es

$= 3113510400 - \dfrac{2!\times 13!}{13\times 2}$ (A2) por fórmula correcta

$= 2634508800$ A1

[3]

Sección B

10. (a) $\mathrm{P}(L > 59,2) = 0,12$ — (M1) por enfoque válido

$\mathrm{P}\left(Z > \frac{59,2-\mu}{3,5}\right) = 0,12$ — (A1) por enfoque correcto

$\frac{59,2-\mu}{3,5} = 1,174986791$ — A1

$59,2-\mu = 4,11245377$

$\mu = 55,08754623$

$\mu = 55,1$ — A1

[4]

(b) $\mathrm{P}(L < q) = 0,55$

$\mathrm{P}\left(Z < \frac{q-55,08754623}{3,5}\right) = 0,55$ — (A1) por enfoque correcto

$\frac{q-55,08754623}{3,5} = 0,1256613375$

$q-55,08754623 = 0,4398146813$

$q = 55,52736091$ — A1

$\therefore q = 55,5$ — A1

[3]

(c) (i) $X \sim \mathrm{B}(10; 0,55)$ — (R1) por distribución correcta

$\mathrm{E}(X) = (10)(0,55)$ — (A1) por sustitución

$\mathrm{E}(X) = 5,5$ — A1

(ii) $\mathrm{P}(X > 5) = 1-\mathrm{P}(X \le 5)$ — (M1) por enfoque válido

$\mathrm{P}(X > 5) = 1-0,4955954083$ — A1

$\mathrm{P}(X > 5) = 0,5044045917$

$\mathrm{P}(X > 5) = 0,504$ — A1

[6]

(d) $m\left(\frac{55\%}{55\%+33\%}\right)(0,8) + m\left(\frac{33\%}{55\%+33\%}\right)(1,1)$ — (M1)(A1) por enfoque correcto

$= (949)(1000)$

$0,5m + 0,4125m = 949000$ — A1

$0,9125m = 949000$

$m = 1040000$ — A1

[4]

11. (a) Cuando $0 \le t \le 1$,

$s(t) = \int \pi t \mathrm{d}t$ (M1) por enfoque válido

$s(t) = \frac{\pi}{2}t^2 + C$ (A1) por valor correcto

$s(0) = -\frac{\pi}{2}$

$\therefore \frac{\pi}{2}(0)^2 + C = -\frac{\pi}{2}$

$C = -\frac{\pi}{2}$

$s(1) = \frac{\pi}{2}(1)^2 - \frac{\pi}{2}$

$s(1) = 0$ (A1) por valor correcto

Cuando $1 < t \le 5$,

$s(t) = \int \pi e^{1-t} \mathrm{d}t$ (M1) por enfoque válido

$s(t) = -\pi e^{1-t} + D$ (A1) por valor correcto

$s(1) = 0$

$\therefore -\pi e^{1-1} + D = 0$

$D = \pi$ (A1) por valor correcto

$s(5) = -\pi e^{1-5} + \pi$

$s(5) = -\pi e^{-4} + \pi$

$s(5) = \frac{\pi}{e^4}(e^4 - 1)$ (A1) por valor correcto

$$\therefore s(t) = \begin{cases} \frac{\pi}{2}t^2 - \frac{\pi}{2} & 0 \le t \le 1 \\ -\pi e^{1-t} + \pi & 1 < t \le 5 \\ \frac{\pi}{e^4}(e^4 - 1) & t > 5 \end{cases}$$ A1

[8]

(b) $a(t)=\begin{cases}\pi(1) & 0\le t\le 1\\ \pi e^{1-t}(-1) & 1<t\le 5\\ 0 & \text{en otro caso}\end{cases}$ (M1) por enfoque válido

$a(t)=\begin{cases}\pi & 0\le t\le 1\\ -\pi e^{1-t} & 1<t\le 5\\ 0 & \text{en otro caso}\end{cases}$ (A1) por valores correctos

$a(t)<-3$

$-\pi e^{1-t}<-3$ (M1) por inecuación

$3-\pi e^{1-t}<0$

Considerando el gráfico de $y=3-\pi e^{1-t}$,

$t<1,0461176$.

$\therefore 1<t<1,05$ A1

[4]

(c) (i) $\frac{ds}{dt}=\pi e^{1-t}$

$\frac{dv}{dt}=-\pi e^{1-t}$

$\therefore \frac{ds}{dv}$

$=\frac{ds}{dt}\div\frac{dv}{dt}$ M1

$=\frac{\pi e^{1-t}}{-\pi e^{1-t}}$ A1

$=-1$ AG

(ii) $\frac{dt}{dv}$

$=1\div\frac{dv}{dt}$ M1

$=\frac{1}{-\pi e^{1-t}}$ A1

$=-\frac{1}{\pi}e^{t-1}$ AG

[4]

12. (a) $|z|$

$= \left|\dfrac{\frac{4}{5}e^{i\theta}}{2}\right|$ (A1) por enfoque correcto

$= \left|\frac{2}{5}e^{i\theta}\right|$

$= \frac{2}{5}\left|e^{i\theta}\right|$

$= \frac{2}{5}(1)$

$= \frac{2}{5}$ A1

[2]

(b) $\dfrac{2}{1-\frac{2}{5}e^{i\theta}}$ A2

[2]

(c) (i) $\dfrac{2}{1-\frac{2}{5}e^{i\theta}}$

$= \dfrac{10}{5-2e^{i\theta}}$

$= \dfrac{10(5-2e^{i(-\theta)})}{(5-2e^{i\theta})(5-2e^{i(-\theta)})}$ M1A1

$= \dfrac{50-20e^{i(-\theta)}}{25-10e^{i(-\theta)}-10e^{i\theta}+4}$ M1

$= \dfrac{50-20e^{i(-\theta)}}{29-10(e^{i(-\theta)}+e^{i\theta})}$

$= \dfrac{50-20(\cos(-\theta)+i\,\text{sen}\,(-\theta))}{29-10(\cos(-\theta)+i\,\text{sen}\,(-\theta)+\cos\theta+i\,\text{sen}\,\theta)}$ A1

$= \dfrac{50-20(\cos\theta-i\,\text{sen}\,\theta)}{29-10(\cos\theta-i\,\text{sen}\,\theta+\cos\theta+i\,\text{sen}\,\theta)}$

$= \dfrac{50-20\cos\theta+20i\,\text{sen}\,\theta}{29-10(2\cos\theta)}$ M1

$= \dfrac{(50-20\cos\theta)+i(20\,\text{sen}\,\theta)}{29-20\cos\theta}$ A1

$$\frac{4}{5}\operatorname{sen}\theta+\frac{8}{25}\operatorname{sen}2\theta+\frac{16}{125}\operatorname{sen}3\theta+\cdots$$

$$=\frac{20\operatorname{sen}\theta}{29-20\cos\theta}$$ M1A1

$$\therefore \operatorname{sen}\theta+\frac{2}{5}\operatorname{sen}2\theta+\frac{4}{25}\operatorname{sen}3\theta+\cdots$$

$$=\frac{25\operatorname{sen}\theta}{29-20\cos\theta}$$ AG

(ii) $$2+\frac{4}{5}\cos\theta+\frac{8}{25}\cos 2\theta+\cdots=\frac{50-20\cos\theta}{29-20\cos\theta}$$ M1A1

$$\frac{4}{5}\cos\theta+\frac{8}{25}\cos 2\theta+\frac{16}{125}\cos 3\theta+\cdots$$

$$=\frac{50-20\cos\theta}{29-20\cos\theta}-2$$ M1

$$=\frac{50-20\cos\theta}{29-20\cos\theta}-\frac{2(29-20\cos\theta)}{29-20\cos\theta}$$

$$=\frac{50-20\cos\theta}{29-20\cos\theta}-\frac{58-40\cos\theta}{29-20\cos\theta}$$ A1

$$=\frac{50-20\cos\theta-58+40\cos\theta}{29-20\cos\theta}$$ M1

$$=\frac{-8+20\cos\theta}{29-20\cos\theta}$$ A1

$$\therefore \cos\theta+\frac{2}{5}\cos 2\theta+\frac{4}{25}\cos 3\theta+\cdots$$

$$=\frac{-10+25\cos\theta}{29-20\cos\theta}$$ A1

$$=\frac{5(-2+5\cos\theta)}{29-20\cos\theta}$$ AG

[15]

Solución de Práctica de Prueba 3 de AE NS Set 2

1. (a) arc P_1B

$= \frac{1}{4}\pi(1)^2$ (M1) por enfoque válido

$= \frac{1}{4}\pi$ A1

[2]

(b) (i) 1 A1

(ii) $\sqrt{2}$ A1

[2]

(c) (i) $R(3)$

$= 3\left(\frac{1}{2}(\text{OA})(\text{OP}_1)\,\text{sen}\,\text{A}\hat{\text{O}}\text{P}_1\right)$ (M1) por enfoque válido

$= 3\left(\frac{1}{2}(1)(1)\,\text{sen}\,\frac{180^\circ}{3}\right)$ (A1) por sustitución

$= \frac{3}{2}\,\text{sen}\,60^\circ$ A1

(ii) $\text{AP}_1 = \text{OP}_1$ ya que AOP_1 es un triángulo equilátero. R1

[4]

(d) $R(4)$

$= 4\left(\frac{1}{2}(\text{OA})(\text{OP}_1)\,\text{sen}\,\text{A}\hat{\text{O}}\text{P}_1\right)$ M1

$= 4\left(\frac{1}{2}(1)(1)\,\text{sen}\,\frac{180^\circ}{4}\right)$ A1

$= 2\,\text{sen}\,45^\circ$ AG

[2]

(e) $AP_1^2 = OA^2 + OP_1^2 - 2(OA)(OP_1)\cos A\hat{O}P_1$ M1

$L(4)^2 = 1^2 + 1^2 - 2(1)(1)\cos 45^\circ$ A1

$L(4)^2 = 2 - 2\left(\frac{\sqrt{2}}{2}\right)$

$L(4)^2 = 2 - \sqrt{2}$ A1

$\because L(4)^4 - 4L(4)^2 + 2$

$= (2-\sqrt{2})^2 - 4(2-\sqrt{2}) + 2$ M1

$= 4 - 4\sqrt{2} + 2 - 8 + 4\sqrt{2} + 2$ M1

$= 0$

Por lo tanto, el valor exacto de $L(4)$ satisface la ecuación $x^4 - 4x^2 + 2 = 0$. AG

[5]

(f) (i) $R(n)$

$= n\left(\frac{1}{2}(OA)(OP_1)\operatorname{sen} A\hat{O}P_1\right)$ M1

$= n\left(\frac{1}{2}(1)(1)\operatorname{sen}\frac{180^\circ}{n}\right)$ A1

$= \frac{n}{2}\operatorname{sen}\frac{180^\circ}{n}$ A1

(ii) $\frac{1}{2}\pi$ A1

[4]

(g) (i) $AP_1^2 = OA^2 + OP_1^2 - 2(OA)(OP_1)\cos A\hat{O}P_1$ M1

$L(n)^2 = 1^2 + 1^2 - 2(1)(1)\cos\frac{180^\circ}{n}$ A1

$L(n)^2 = 2 - 2\cos\frac{180^\circ}{n}$

$L(n) = \sqrt{2 - 2\cos\frac{180^\circ}{n}}$ AG

(ii) $\frac{L(n)}{R(n)}$

$$=\frac{\sqrt{2-2\cos\frac{180^\circ}{n}}}{\frac{n}{2}\operatorname{sen}\frac{180^\circ}{n}}$$

$$=\frac{\sqrt{2-2\left(1-2\operatorname{sen}^2\frac{90^\circ}{n}\right)}}{\frac{n}{2}\left(2\operatorname{sen}\frac{90^\circ}{n}\cos\frac{90^\circ}{n}\right)}$$ A2

$$=\frac{\sqrt{4\operatorname{sen}^2\frac{90^\circ}{n}}}{n\operatorname{sen}\frac{90^\circ}{n}\cos\frac{90^\circ}{n}}$$ M1

$$=\frac{2\operatorname{sen}\frac{90^\circ}{n}}{n\operatorname{sen}\frac{90^\circ}{n}\cos\frac{90^\circ}{n}}$$ M1

$$=\frac{2}{n\cos\frac{90^\circ}{n}}$$

$$=\frac{2}{n}\sec\frac{90^\circ}{n}$$ AG

[6]

(h) $\frac{L(n)}{R(n)}<\frac{1}{\pi^\pi}$

$\therefore \frac{2}{n}\sec\frac{90^\circ}{n}<\frac{1}{\pi^\pi}$

$\frac{2}{n}\sec\frac{90^\circ}{n}-\frac{1}{\pi^\pi}<0$ (A1) por inecuación correcta

Considerando la gráfica de $y=\frac{2}{n}\sec\frac{90^\circ}{n}-\frac{1}{\pi^\pi}$,

$n>72.941232$.

Por lo tanto, el valor mínimo de n es 73. A1

[2]

2. (a) $f'(x)$

$= (e^x)(1-x)^n + (e^x)(n)(1-x)^{n-1}(-1)$ A1

$= e^x(1-x)^{n-1}\left[(1-x)+n(-1)\right]$

$= e^x(1-x)^{n-1}(1-x-n)$ A1

$e^x > 0$, $(1-x)^{n-1} > 0$ y $1-x-n < 0$ para $n > 0$. R1

$\therefore f'(x) < 0$

Por lo tanto, $f(x)$ decrece en $0 < x < 1$ para $n > 0$. AG

[3]

(b) $f(0) = 1$ y $f(1) = 0$. R1

También, $f(x)$ decrece en $0 < x < 1$.

Por lo tanto, el área bajo la gráfica $f(x)$ es positiva, y es más pequeña que el área del cuadrado de longitud 1. R1

Por lo tanto, $0 < I(n) < 1$ para $n > 0$. AG

[2]

(c) (i) $I(0)$

$= \int_0^1 e^x(1-x)^0 \mathrm{d}x$ M1

$= \int_0^1 e^x \mathrm{d}x$

$= \left[e^x\right]_0^1$ A1

$= e^1 - e^0$

$= e - 1$ AG

(ii) $I(1)$

$= \int_0^1 e^x (1-x)^1 \, \mathrm{d}x$

Sea $\theta = e^x$. (M1) por enfoque válido

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = e^x$

$\therefore I(1)$

$= \int_0^1 (1-x) \cdot \frac{\mathrm{d}(e^x)}{\mathrm{d}x} \mathrm{d}x$

$= \left[(1-x)e^x\right]_0^1 - \int_0^1 e^x \cdot \frac{\mathrm{d}(1-x)}{\mathrm{d}x} \mathrm{d}x$ A1

$= \left[(1-x)e^x\right]_0^1 - \int_0^1 e^x (-1) \mathrm{d}x$ A1

$= \left[(1-x)e^x\right]_0^1 + \int_0^1 e^x \mathrm{d}x$

$= \left[(1-x)e^x\right]_0^1 + e - 1$ A1

$= ((1-1)e^1 - (1-0)e^0) + e - 1$

$= (0-1) + e - 1$

$= e - 2$ A1

(iii) $I(2)$

$= \int_0^1 e^x (1-x)^2 \mathrm{d}x$

Sea $\theta = e^x$. (M1) por enfoque válido

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = e^x$

$\therefore I(2)$

$= \int_0^1 (1-x)^2 \cdot \frac{\mathrm{d}(e^x)}{\mathrm{d}x} \mathrm{d}x$

$= \left[(1-x)^2 e^x \right]_0^1 - \int_0^1 e^x \cdot \frac{\mathrm{d}((1-x)^2)}{\mathrm{d}x} \mathrm{d}x$ A1

$= \left[(1-x)^2 e^x \right]_0^1 - \int_0^1 e^x \cdot 2(1-x)(-1) \mathrm{d}x$ A1

$= \left[(1-x)^2 e^x \right]_0^1 + 2\int_0^1 e^x (1-x) \mathrm{d}x$

$= \left[(1-x)^2 e^x \right]_0^1 + 2I(1)$ A1

$= \left[(1-x)^2 e^x \right]_0^1 + 2(e-2)$

$= ((1-1)^2 e^1 - (1-0)^2 e^0) + 2(e-2)$

$= (0-1) + 2e - 4$

$= 2e - 5$ A1

[12]

(d) $I(n)$

$= \int_0^1 e^x (1-x)^n \, \mathrm{d}x$

Sea $\theta = e^x$. M1

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = e^x$

$\therefore I(n)$

$= \int_0^1 (1-x)^n \cdot \frac{\mathrm{d}(e^x)}{\mathrm{d}x} \mathrm{d}x$

$= \left[(1-x)^n e^x\right]_0^1 - \int_0^1 e^x \cdot \frac{\mathrm{d}((1-x)^n)}{\mathrm{d}x} \mathrm{d}x$ A1

$= \left[(1-x)^n e^x\right]_0^1 - \int_0^1 e^x \cdot n(1-x)^{n-1}(-1)\mathrm{d}x$ A1

$= \left[(1-x)^n e^x\right]_0^1 + n\int_0^1 e^x (1-x)^{n-1} \mathrm{d}x$

$= \left[(1-x)^n e^x\right]_0^1 + nI(n-1)$ A1

$= ((1-1)^n e^1 - (1-0)^n e^0) + nI(n-1)$

$= -1 + nI(n-1)$ A1

Por lo tanto, $I(n) = nI(n-1) - 1$ para $n > 0$. AG

[5]

(e) $I(n)$

$= nI(n-1) - 1$

$= n((n-1)I(n-2) - 1) - 1$ M1

$= n(n-1)I(n-2) - n - 1$

$= n(n-1)((n-2)I(n-3) - 1) - n - 1$ M1

$= n(n-1)(n-2)I(n-3) - n(n-1) - n - 1$

$= \cdots$

$= n(n-1)(n-2)\cdots(2)(1)I(0)$

$-n(n-1)(n-2)\cdots(2)$ A1

$-\cdots - n(n-1)(n-2) - n(n-1) - n - 1$

$= n!\begin{bmatrix} I(0) - \frac{1}{1!} - \cdots \\ -\frac{1}{(n-3)!} - \frac{1}{(n-2)!} - \frac{1}{(n-1)!} - \frac{1}{n!} \end{bmatrix}$ M1A1

$= n!\left[e - 1 - \left(\frac{1}{1!} + \cdots + \frac{1}{(n-2)!} + \frac{1}{(n-1)!} + \frac{1}{n!}\right)\right]$

$\therefore I(n) = n!\left[e - 1 - \sum_{r=1}^{n} \frac{1}{r!}\right]$ AG

[5]

(f) e A1

[1]

Solución de Práctica de Prueba 1 de AE NS Set 3

Sección A

1. (a) La diferencia común

$= 95 - 100$ (M1) por enfoque válido

$= -5$ A1

[2]

(b) El décimoquinto término

$= 100 + (15-1)(-5)$ (A1) por sustitución

$= 30$ A1

[2]

(c) La suma de los primeros quince términos

$= \frac{15}{2}\left[2(100) + (15-1)(-5)\right]$ (A1) por sustitución

$= 975$ A1

[2]

2. (a) El gradiente de L_1 es 2. A1

El punto de intersección con el eje y de L_1 es -20. A1

[2]

(b) El gradiente de L_2 es $-\frac{1}{2}$. (A1) por valor correcto

La ecuación de L_2:

$y - (-20) = -\frac{1}{2}(x - 0)$ A1

$y + 20 = -\frac{1}{2}x$

$2y + 40 = -x$

$x + 2y + 40 = 0$ A1

[3]

3. (a) (i) 4 A1

(ii) $\frac{1}{3}$ A1

(iii) -1 A1

[3]

(b) $\log_{27} x+\frac{8}{3}=\log_4 256+\log_{125} 5+\log_{\pi}\frac{1}{\pi}$

$\log_{27} x+\frac{8}{3}=4+\frac{1}{3}-1$ (M1) por sustitución

$\log_{27} x=\frac{2}{3}$

$x=27^{\frac{2}{3}}$ (A1) por enfoque correcto

$x=(3^3)^{\frac{2}{3}}$

$x=3^2$

$x=9$ A1

[3]

4. $\left(1-\frac{3}{4}x\right)^n(1+2nx)^3$

$=\left(1+\binom{n}{1}\left(-\frac{3}{4}x\right)+\cdots\right)\left(1+\binom{3}{1}(2nx)+\cdots\right)$ (M1) por expansión válida

$=\left(1+(n)\left(-\frac{3}{4}x\right)+\cdots\right)(1+(3)(2nx)+\cdots)$ (A1) por enfoque correcto

$=\left(1-\frac{3}{4}nx+\cdots\right)(1+6nx+\cdots)$ A2

El coeficiente de x

$=(1)(6n)+\left(-\frac{3}{4}n\right)(1)$ (A1) por enfoque correcto

$=\frac{21}{4}n$

$\therefore \frac{21}{4}n=\frac{105}{4}$ (M1) por ecuación

$n=5$ A1

[7]

5. $-3\sqrt{3} \le f(x) \le 3\sqrt{3}$

$-3\sqrt{3} \le 6\,\mathrm{sen}\,2x \le 3\sqrt{3}$

$-\frac{\sqrt{3}}{2} \le \mathrm{sen}\,2x \le \frac{\sqrt{3}}{2}$ A1

$\therefore \mathrm{sen}\left(-\frac{\pi}{3}\right) \le \mathrm{sen}\,2x \le \mathrm{sen}\,\frac{\pi}{3}$,

$\mathrm{sen}\left(\pi-\frac{\pi}{3}\right) \le \mathrm{sen}\,2x \le \mathrm{sen}\left(\pi+\frac{\pi}{3}\right)$ o (A2) por rangos correctos

$\mathrm{sen}\left(2\pi-\frac{\pi}{3}\right) \le \mathrm{sen}\,2x \le \mathrm{sen}\left(2\pi+\frac{\pi}{3}\right)$

$-\frac{\pi}{3} \le 2x \le \frac{\pi}{3}$, $\frac{2\pi}{3} \le 2x \le \frac{4\pi}{3}$ o $\frac{5\pi}{3} \le 2x \le \frac{7\pi}{3}$ A1

$-\frac{\pi}{6} \le x \le \frac{\pi}{6}$, $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$ o $\frac{5\pi}{6} \le x \le \frac{7\pi}{6}$ (M1) por enfoque válido

$\therefore 0 \le x \le \frac{\pi}{6}$, $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$ o $\frac{5\pi}{6} \le x \le \frac{7\pi}{6}$ A3

[8]

6. (a) $\mathrm{E}(X)=\int_{-2}^{3} x\cdot\frac{1}{5}\mathrm{d}x$ (A1) por sustitución

$\mathrm{E}(X)=\left[\frac{1}{10}x^2\right]_{-2}^{3}$

$\mathrm{E}(X)=\frac{9}{10}-\frac{4}{10}$

$\mathrm{E}(X)=\frac{1}{2}$ A1

[2]

(b) $\mathrm{E}(X^2)=\int_{-2}^{3} x^2\cdot\frac{1}{5}\mathrm{d}x$ (A1) por sustitución

$\mathrm{E}(X^2)=\left[\frac{1}{15}x^3\right]_{-2}^{3}$

$\mathrm{E}(X^2)=\frac{27}{15}-\left(-\frac{8}{15}\right)$

$\mathrm{E}(X^2)=\frac{7}{3}$ A1

[2]

(c) La desviación típica

$=\sqrt{\mathrm{E}(X^2)-(\mathrm{E}(X))^2}$ (A1) por sustitución

$=\sqrt{\frac{7}{3}-\left(\frac{1}{2}\right)^2}$

$=\sqrt{\frac{25}{12}}$ A1

[2]

7. (a) $f(|-x|)=\frac{7-2|-x|}{3-|-x|}$ M1

$f(|-x|)=\frac{7-2|x|}{3-|x|}$ A1

$f(|-x|)=f(|x|)$

Por lo tanto, $f(|x|)$ es una función par. AG

[2]

(b) $x=3$, $x=-3$ A2

[2]

(c) $y=-2$ A1

[1]

8. $2(\sec\alpha + 2\tan\alpha)^2 = 3 + 8\sec\alpha\tan\alpha + 6\tan^2\alpha$

$2(\sec^2\alpha + 4\sec\alpha\tan\alpha + 4\tan^2\alpha)$
$= 3 + 8\sec\alpha\tan\alpha + 6\tan^2\alpha$ (M1) por enfoque válido

$2\sec^2\alpha + 8\sec\alpha\tan\alpha + 8\tan^2\alpha$
$= 3 + 8\sec\alpha\tan\alpha + 6\tan^2\alpha$

$2\sec^2\alpha + 2\tan^2\alpha = 3$

$\sec^2\alpha + \tan^2\alpha = \frac{3}{2}$

$1 + \tan^2\alpha + \tan^2\alpha = \frac{3}{2}$ A1

$2\tan^2\alpha = \frac{1}{2}$

$\tan^2\alpha = \frac{1}{4}$

$\tan\alpha = -\frac{1}{2}$ o $\tan\alpha = \frac{1}{2}$ (*Rechazada*) (A1) por valor correcto

$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$ (M1) por enfoque válido

$\tan 2\alpha = \frac{2\left(-\frac{1}{2}\right)}{1 - \left(-\frac{1}{2}\right)^2}$

$\tan 2\alpha = -\frac{4}{3}$ A1

[5]

9. Cuando $n=1$,

$1^3+3(1)^2-1=3$

$1^3+3(1)^2-1=3(1)$ A1

Por tanto, el enunciado es cierto para $n=1$.

Suponemos que es cierto para $n=k$. M1

$k^3+3k^2-k=3M$, donde $M\in\mathbb{Z}$.

Cuando $n=k+1$,

$(k+1)^3+3(k+1)^2-(k+1)$

$=k^3+3k^2+3k+1+3(k^2+2k+1)-k-1$ M1

$=(3M+k-3k^2)+3k^2+3k+1+3k^2+6k+3-k-1$ A1

$=3M+3k^2+9k+3$ M1

$=3(M+k^2+3k+1)$, donde $M+k^2+3k+1\in\mathbb{Z}$. A1

Por tanto, el enunciado es cierto cuando $n=k+1$.

Por lo tanto, el enunciado es cierto para todo $n\in\mathbb{Z}^+$. R1

[7]

Sección B

10. (a) $g(x)-f(x)=0$

$e^{\frac{1}{2}\sqrt{x}}-e^{\frac{1}{2}\sqrt{x}}\operatorname{sen}\left(\frac{\pi}{3}x\right)=0$ (M1) por enfoque válido

$e^{\frac{1}{2}\sqrt{x}}\left(1-\operatorname{sen}\left(\frac{\pi}{3}x\right)\right)=0$

$1-\operatorname{sen}\left(\frac{\pi}{3}x\right)=0$

$\operatorname{sen}\left(\frac{\pi}{3}x\right)=1$ A1

$\frac{\pi}{3}x=\frac{\pi}{2}$, $\frac{\pi}{3}x=\frac{5\pi}{2}$ o $\frac{\pi}{3}x=\frac{9\pi}{2}$ (A1) por valores correctos

$x=\frac{3}{2}$, $x=\frac{15}{2}$ o $x=\frac{27}{2}$ A3

[6]

(b) (i) $\frac{\pi}{3}x_n=\frac{\pi}{2}+(n-1)(2\pi)$ A1

$x_n=\frac{3}{2}+6(n-1)$

$x_{n+1}-x_n$

$=\left(\frac{3}{2}+6((n+1)-1)\right)-\left(\frac{3}{2}+6(n-1)\right)$ M1

$x_{n+1}-x_n=\left(\frac{3}{2}+6n\right)-\left(\frac{3}{2}+6n-6\right)$

$x_{n+1}-x_n=6$ A1

Las diferencias entre cada par de términos consecutivos son todas 6. Luego, $x_1, x_2, x_3, \ldots$ es una progresión aritmética. AG

(ii) $x_n=\frac{3}{2}+6n-6$

$x_n=6n-\frac{9}{2}$ A1

[4]

(c) Nótese que $x_2 = \dfrac{15}{2}$ y $x_3 = \dfrac{27}{2}$.

$f(x) = 0$

$e^{\frac{1}{2}\sqrt{x}} \operatorname{sen}\left(\dfrac{\pi}{3}x\right) = 0$ M1

$\operatorname{sen}\left(\dfrac{\pi}{3}x\right) = 0$

$\dfrac{\pi}{3}x = 3\pi$ o $\dfrac{\pi}{3}x = 4\pi$

$x = 9$ o $x = 12$ (A1) por valores correctos

$\therefore R = \int_{\frac{15}{2}}^{9}\left(e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}} \operatorname{sen}\left(\dfrac{\pi}{3}x\right)\right)\mathrm{d}x + \int_{9}^{12} e^{\frac{1}{2}\sqrt{x}}\,\mathrm{d}x$

$+\int_{12}^{\frac{27}{2}}\left(e^{\frac{1}{2}\sqrt{x}} - e^{\frac{1}{2}\sqrt{x}} \operatorname{sen}\left(\dfrac{\pi}{3}x\right)\right)\mathrm{d}x$ A2

[4]

11. (a)

$\vec{PS} = \vec{PQ} + \vec{QS}$

$\vec{PS} = \mathbf{r} + \frac{1}{\alpha+1}\vec{QR}$ (A1) por enfoque correcto

$\vec{PS} = \mathbf{r} + \frac{1}{\alpha+1}(\vec{PR} - \vec{PQ})$ (M1) por enfoque válido

$\vec{PS} = \mathbf{r} + \frac{1}{\alpha+1}(\mathbf{q} - \mathbf{r})$

$\vec{PS} = \frac{\alpha+1}{\alpha+1}\mathbf{r} + \frac{1}{\alpha+1}\mathbf{q} - \frac{1}{\alpha+1}\mathbf{r}$ (M1) por enfoque válido

$\vec{PS} = \frac{1}{\alpha+1}\mathbf{q} + \frac{\alpha}{\alpha+1}\mathbf{r}$ A1

[4]

(b)

$\because \mathrm{PS} \perp \mathrm{QR}$

$\therefore \vec{PS} \cdot \vec{QR} = 0$ M1

$\left(\frac{1}{\alpha+1}\mathbf{q} + \frac{\alpha}{\alpha+1}\mathbf{r}\right) \cdot (\mathbf{q} - \mathbf{r}) = 0$ A1

$(\mathbf{q} + \alpha\mathbf{r}) \cdot (\mathbf{q} - \mathbf{r}) = 0$

$\mathbf{q} \cdot (\mathbf{q} - \mathbf{r}) + \alpha\mathbf{r} \cdot (\mathbf{q} - \mathbf{r}) = 0$ M1

$\alpha\mathbf{r} \cdot (\mathbf{q} - \mathbf{r}) = -\mathbf{q} \cdot (\mathbf{q} - \mathbf{r})$

$\alpha\mathbf{r} \cdot (\mathbf{q} - \mathbf{r}) = \mathbf{q} \cdot (\mathbf{r} - \mathbf{q})$ M1

$\alpha = \frac{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}$ AG

[4]

(c)

$\vec{PS} = \frac{1}{\alpha+1}\mathbf{q} + \frac{\alpha}{\alpha+1}\mathbf{r}$

$\vec{PS} = \frac{1}{\alpha+1}(\mathbf{q} + \alpha\mathbf{r})$

$\vec{PS} = \frac{1}{\frac{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})} + 1}\left(\mathbf{q} + \frac{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}\mathbf{r}\right)$ M1

$\vec{PS} = \frac{1}{\frac{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})} + \frac{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}}\left(\frac{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}\mathbf{q} + \frac{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}\mathbf{r}\right)$ M1

$\vec{PS} = \frac{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q}) + \mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}\left(\frac{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}\mathbf{q} + \frac{\mathbf{q} \cdot (\mathbf{r} - \mathbf{q})}{\mathbf{r} \cdot (\mathbf{q} - \mathbf{r})}\mathbf{r}\right)$ A1

$$\overrightarrow{PS} = \frac{(\mathbf{r}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{r}}{\mathbf{q}\cdot(\mathbf{r}-\mathbf{q})+\mathbf{r}\cdot(\mathbf{q}-\mathbf{r})}$$ M1

$$\overrightarrow{PS} = \frac{(\mathbf{r}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{r}}{\mathbf{q}\cdot(\mathbf{r}-\mathbf{q})-\mathbf{r}\cdot(\mathbf{r}-\mathbf{q})}$$ A1

$$\overrightarrow{PS} = \frac{(\mathbf{r}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{r}}{(\mathbf{q}-\mathbf{r})\cdot(\mathbf{r}-\mathbf{q})}$$

$$\overrightarrow{PS} = -\frac{(\mathbf{r}\cdot(\mathbf{q}-\mathbf{r}))\mathbf{q}+(\mathbf{q}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{r}}{|\mathbf{r}-\mathbf{q}|^2}$$ A1

$$\overrightarrow{PS} = \frac{(\mathbf{r}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{q}-(\mathbf{q}\cdot(\mathbf{r}-\mathbf{q}))\mathbf{r}}{|\mathbf{r}-\mathbf{q}|^2}$$ AG

[6]

(d) (i)

$$\alpha = \frac{\mathbf{q}\cdot(\mathbf{r}-\mathbf{q})}{\mathbf{r}\cdot(\mathbf{q}-\mathbf{r})}$$

$$\alpha = \frac{\mathbf{q}\cdot\mathbf{r}-\mathbf{q}\cdot\mathbf{q}}{\mathbf{r}\cdot\mathbf{q}-\mathbf{r}\cdot\mathbf{r}}$$ (M1) por enfoque válido

$$\alpha = \frac{\mathbf{q}\cdot\mathbf{r}-|\mathbf{q}|^2}{\mathbf{r}\cdot\mathbf{q}-|\mathbf{r}|^2}$$

$$\alpha = \frac{0-20^2}{0-15^2}$$ (A1) por sustitución

$$\alpha = \frac{16}{9}$$ A1

(ii)

$$QR = \sqrt{20^2+15^2}$$

$$QR = 25$$ (A1) por valor correcto

$$RS = 25\left(\frac{\frac{16}{9}}{\frac{16}{9}+1}\right)$$

$$RS = 16$$ (A1) por valor correcto

$$PS = \sqrt{20^2-16^2}$$

$$PS = 12$$ (A1) por valor correcto

El área requerida

$$= \frac{(16)(12)}{2}$$

$$= 96$$ A1

[7]

12. (a)

$R^2 = \mathrm{OP}^2 + r^2$	(M1) por enfoque válido
$R^2 = (h-R)^2 + r^2$	
$R^2 = h^2 - 2Rh + R^2 + r^2$	
$2Rh - h^2 = r^2$	A1
$V = \frac{1}{3}\pi r^2 h$	
$\therefore V = \frac{1}{3}\pi(2Rh - h^2)(h)$	(A1) por sustitución
$\therefore V = \frac{2R}{3}\pi h^2 - \frac{1}{3}\pi h^3$	A1

[4]

(b)

$V = \frac{2R}{3}\pi h^2 - \frac{1}{3}\pi h^3$	
$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2R}{3}\pi(2h) - \frac{1}{3}\pi(3h^2)$	M1A1
$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{4R}{3}\pi h - \pi h^2$	A1
$\frac{\mathrm{d}^2V}{\mathrm{d}h^2} = \frac{4R}{3}\pi(1) - \pi(2h)$	A1
$\frac{\mathrm{d}^2V}{\mathrm{d}h^2} = \frac{4R}{3}\pi - 2\pi h$	AG

[4]

(c) $\frac{dV}{dh}=0$

$\therefore \frac{4R}{3}\pi h-\pi h^2=0$ M1

$\frac{4R}{3}-h=0$

$h=\frac{4R}{3}$ A1

$\left.\frac{d^2V}{dh^2}\right|_{h=\frac{4R}{3}}=\frac{4R}{3}\pi-2\pi\left(\frac{4R}{3}\right)$ M1

$\left.\frac{d^2V}{dh^2}\right|_{h=\frac{4R}{3}}=-\frac{4R}{3}\pi$

$\left.\frac{d^2V}{dh^2}\right|_{h=\frac{4R}{3}}<0$ R1

Por lo tanto, V alcanza el valor máximo cuando $h=\frac{4R}{3}$.

$2R\left(\frac{4R}{3}\right)-\left(\frac{4R}{3}\right)^2=r^2$ A1

$\frac{8R^2}{3}-\frac{16R^2}{9}=r^2$ M1

$\frac{8R^2}{9}=r^2$

$r=\frac{2\sqrt{2}R}{3}$

Por lo tanto, V alcanza su máximo cuando $r=\frac{2\sqrt{2}R}{3}$. AG

[6]

(d) $\frac{32}{81}\pi R^3$ A2

[2]

(e) La generatriz del cono circular

$= \sqrt{\left(\frac{2\sqrt{2}R}{3}\right)^2 + \left(\frac{4R}{3}\right)^2}$ (M1) por enfoque válido

$= \sqrt{\frac{24}{9}R^2}$

$= \frac{\sqrt{24}R}{3}$

$= \frac{2\sqrt{6}R}{3}$ A1

El área de la superficie curva del cono circular

$= \pi\left(\frac{2\sqrt{2}R}{3}\right)\left(\frac{2\sqrt{6}R}{3}\right)$

$= \frac{4}{9}\sqrt{12}\pi R^2$

$< \frac{4}{9}(4)\pi R^2$ R1

$= \frac{16}{9}\pi R^2$

Por lo tanto, el área de la superficie curva del cono no es mayor que $\frac{16}{9}\pi R^2$ cuando su volumen alcanza su máximo. A1

[4]

Solución de Práctica de Prueba 2 de AE NS Set 3

Sección A

1.	(a)	(i)	6	A1
		(ii)	6	A1
		(iii)	El rango	
			$=18-3$	(M1) por enfoque válido
			$=15$	A1
				[4]
	(b)	(i)	La media	
			$=\dfrac{(3)(12)+(6)(20)+(9)(12)+(12)(8)+(15)(4)+(18)(4)}{12+20+12+8+4+4}$	(M1) por enfoque válido
			$=8{,}2$	A1
		(ii)	La varianza	
			$=4{,}308131846^2$	(M1) por enfoque válido
			$=18{,}6$	A1
				[4]

2. (a) $f(x) = g(x)$

$\pi e^{-x^2} = 1 + \dfrac{1}{\pi e^{-x^2}}$ (M1) por ecuación

$\pi e^{-x^2} - 1 - \dfrac{e^{x^2}}{\pi} = 0$

Considerando el gráfico de $y = \pi e^{-x^2} - 1 - \dfrac{e^{x^2}}{\pi}$,

$x = -0,814566$ o $x = 0,8145662$.

$\therefore a = -0,815$, $b = 0,815$ A2

[3]

(b) El área requerida

$= \int_{-0,814566}^{0,8145662} (f(x) - g(x))\mathrm{d}x$ (A1) por integral correcta

$= \int_{-0,814566}^{0,8145662} \left(\pi e^{-x^2} - 1 - \dfrac{e^{x^2}}{\pi} \right) \mathrm{d}x$

$= 1,890606422$ (A1) por valor correcto

$= 1,89$ A1

[3]

3. Observe que $f(0) = -1$.

$-1 = \sqrt{2}\,\mathrm{sen}\left(\dfrac{\pi}{6}(0+h) \right)$ (M1) por ecuación

$-\dfrac{1}{\sqrt{2}} = \mathrm{sen}\left(\dfrac{\pi}{6}h \right)$ (A1) por enfoque correcto

$\dfrac{\pi}{6}h = -\dfrac{3\pi}{4}$ o $\dfrac{\pi}{6}h = -\dfrac{\pi}{4}$ (A1) por enfoque correcto

$h = -4,5$ (*Rechazada*) o $h = -1,5$ A1

$\therefore h = -1,5$ A1

[5]

4. (a) (i) $\frac{1}{2}$ A1

(ii) 3 A1

(iii) -4 A1

[3]

(b) Las coordenadas de P'

$=\left(\frac{2}{2}+3,-5(8-4)\right)$ (A2) por enfoque correcto

$=(4,-20)$ A2

[4]

5. (a) $\cos\theta=\frac{\text{AB}}{r}$

$\text{AB}=r\cos\theta$ A1

[1]

(b) $\text{sen}\,\theta=\frac{\text{AE}}{r}$

$\text{AE}=r\,\text{sen}\,\theta$ A1

El área del triángulo ABE

$=\frac{(\text{AB})(\text{AE})}{2}$

$=\frac{(r\cos\theta)(r\,\text{sen}\,\theta)}{2}$ M1

$=\frac{1}{2}r^2\text{sen}\,\theta\cos\theta$ A1

$=\frac{1}{2}r^2\left(\frac{1}{2}\text{sen}\,2\theta\right)$ A1

$=\frac{r^2\text{sen}\,2\theta}{4}$ AG

[4]

(c) $\text{A}\hat{\text{E}}\text{B}+\text{B}\hat{\text{E}}\text{C}+\text{C}\hat{\text{E}}\text{D}=\pi$ M1

$\left(\frac{\pi}{2}-\theta\right)+\text{B}\hat{\text{E}}\text{C}+\left(\frac{\pi}{2}-\theta\right)=\pi$ A1

$\pi-2\theta+\text{B}\hat{\text{E}}\text{C}=\pi$

$\text{B}\hat{\text{E}}\text{C}=2\theta$ AG

[2]

6. (a) Sea $\frac{x^2+2x+4}{(x-3)(x-7)} \equiv A + \frac{B}{x-3} + \frac{C}{x-7}$, donde A, B y C son constantes.

$$\frac{x^2+2x+4}{(x-3)(x-7)} \equiv \frac{A(x-3)(x-7)}{(x-3)(x-7)} + \frac{B(x-7)}{(x-3)(x-7)} + \frac{C(x-3)}{(x-3)(x-7)}$$ M1

$$\frac{x^2+2x+4}{(x-3)(x-7)} \equiv \frac{Ax^2-10Ax+21A+Bx-7B+Cx-3C}{(x-3)(x-7)}$$

$$x^2+2x+4 \equiv Ax^2+(-10A+B+C)x+(21A-7B-3C)$$ A1

$A=1$ A1

$2=-10(1)+B+C$

$C=12-B$

$4=21A-7B-3C$

$\therefore 4=21(1)-7B-3(12-B)$ A1

$4=21-7B-36+3B$

$19=-4B$

$B=-\frac{19}{4}$ A1

$\therefore C=12-\left(-\frac{19}{4}\right)$

$C=\frac{67}{4}$ A1

[6]

(b) $y=1$ A1

[1]

7. (a) (i) $\begin{cases} x+2y-z=1 \\ 2x-y+az=0 \\ x+3y+2z=b \end{cases}$

$\rightarrow \begin{cases} x+2y-z=1 \\ -5y+(a+2)z=-2 \\ y+3z=b-1 \end{cases}$ M1

$(R_2-2R_1 \,\&\, R_3-R_1)$

$\rightarrow \begin{cases} x+2y-z=1 \\ -5y+(a+2)z=-2 \quad (R_3+0,2R_2) \\ (0,2a+3,4)z=b-1,4 \end{cases}$ A1

El Sistema no tiene solución cuando

$0,2a+3,4=0$ y $b-1,4\neq 0$.

$a=-17$ y $b\neq 1,4$ A1

(ii) El Sistema tiene solución única cuando

$0,2a+3,4\neq 0$.

$\therefore a\neq -17$ y $b\in\mathbb{R}$ A1

[4]

(b) $\begin{cases} x+2y-z=1 \\ 2x-y+3z=0 \\ x+3y+2z=3 \end{cases}$

Resolviendo el sistema, $x=-0,2$, $y=0,8$ y

$z=0,4$. A2

[2]

8. $\mathbf{r}=(-1+2\lambda+4\mu)\mathbf{i}+(3+\lambda)\mathbf{j}+(-1+5\mu)\mathbf{k}$

$\mathbf{r}=(-\mathbf{i}+3\mathbf{j}-\mathbf{k})+\lambda(2\mathbf{i}+\mathbf{j})+\mu(4\mathbf{i}+5\mathbf{k})$ (M1) por enfoque válido

$\mathbf{n}=(2\mathbf{i}+\mathbf{j})\times(4\mathbf{i}+5\mathbf{k})$

$\mathbf{n}=\begin{pmatrix}(1)(5)-(0)(0)\\(0)(4)-(2)(5)\\(2)(0)-(1)(4)\end{pmatrix}$

$\mathbf{n}=5\mathbf{i}-10\mathbf{j}-4\mathbf{k}$ (A1) por valores correctos

La ecuación cartesiana del plano π:

$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot(5\mathbf{i}-10\mathbf{j}-4\mathbf{k})=(-\mathbf{i}+3\mathbf{j}-\mathbf{k})\cdot(5\mathbf{i}-10\mathbf{j}-4\mathbf{k})$ M1A1

$5x-10y-4z=(-1)(5)+(3)(-10)+(-1)(-4)$

$5x-10y-4z=-31$ A1

[5]

9. (a) $f(0)=\arctan\frac{\pi}{2}(0)=0$ (A1) por valor correcto

$$f'(x)=\left(\frac{1}{1+\left(\frac{\pi}{2}x\right)^2}\right)\left(\frac{\pi}{2}\right)$$

$$f'(x)=\frac{2\pi}{4+\pi^2x^2}$$

$$f'(0)=\frac{2\pi}{4+\pi^2(0)^2}=\frac{\pi}{2}$$ (A1) por valor correcto

$$f''(x)=\frac{(4+\pi^2x^2)(0)-(2\pi)(2\pi^2x)}{(4+\pi^2x^2)^2}$$ (M1) por enfoque válido

$$f''(x)=-\frac{4\pi^3x}{(4+\pi^2x^2)^2}$$

$$f''(0)=-\frac{4\pi^3(0)}{(4+\pi^2(0)^2)^2}=0$$ (A1) por valor correcto

$$f^{(3)}(x)=-\frac{(4+\pi^2x^2)^2(4\pi^3)-(4\pi^3x)(2)(4+\pi^2x^2)(2\pi^2x)}{(4+\pi^2x^2)^4}$$ (M1) por enfoque válido

$$f^{(3)}(0)=-\frac{(4+0)^2(4\pi^3)-0}{4^4}=-\frac{\pi^3}{4}$$ (A1) por valor correcto

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f^{(3)}(0)+\cdots$$

$$f(x)=0+x\left(\frac{\pi}{2}\right)+\frac{x^2}{2}(0)+\frac{x^3}{6}\left(-\frac{\pi^3}{4}\right)+\cdots$$

$$f(x)=\frac{\pi}{2}x-\frac{\pi^3}{24}x^3+\cdots$$ A1

[7]

Sección B

10. (a) $a = -0,0807147258$

$a = -0,0807$ A1

$b = 3,177202711$

$b = 3,18$ A1

[2]

(b) $\log y = -0,0807147258\sqrt{9} + 3,177202711$ (M1) por enfoque válido

$\log y = 2,935058534$

$y = 10^{2,935058534}$ (M1) por enfoque válido

$y = 861,1098035$

$y = 861$ A1

[3]

(c) $\log y = -0,0807147258\sqrt{x} + 3,177202711$

$y = 10^{-0,0807147258\sqrt{x}+3,177202711}$ (M1) por enfoque válido

$y = 10^{-0,0807147258\sqrt{x}} \cdot 10^{3,177202711}$ (A1) por enfoque correcto

$y = 10^{3,177202711} \cdot (10^{-0,0807147258})^{\sqrt{x}}$ A1

$k = 10^{3,177202711}$ (A1) por enfoque correcto

$k = 1503,843735$

$k = 1500$ A1

$m = 10^{-0,0807147258}$ (A1) por enfoque correcto

$m = 0,8303960491$

$m = 0,830$ A1

[7]

11. (a) $a = \frac{v^2 + 64}{240}$

$\frac{dv}{dt} = \frac{v^2 + 64}{240}$

$\frac{1}{v^2 + 64} dv = \frac{1}{240} dt$ (M1) por enfoque válido

$\int \frac{1}{v^2 + 64} dv = \int \frac{1}{240} dt$ (A1) por enfoque correcto

$\frac{1}{8} \arctan \frac{v}{8} = \frac{1}{240} t + C$ A1

$\arctan \frac{v}{8} = \frac{1}{30} t + C$

$\frac{v}{8} = \tan\left(\frac{1}{30} t + C\right)$

$v = 8 \tan\left(\frac{1}{30} t + C\right)$ A1

$0 = 8 \tan\left(\frac{1}{30}(0) + C\right)$ (M1) por sustitución

$C = 0$ (A1) por valor correcto

$\therefore v = 8 \tan \frac{1}{30} t$ A1

[7]

(b) $\arctan \frac{v}{8} = \frac{1}{30} t$

$\arctan\left(\frac{1}{8} \cdot \frac{8}{3}\sqrt{3}\right) = \frac{1}{30} t$ (M1) por ecuación

$\arctan \frac{\sqrt{3}}{3} = \frac{1}{30} t$

$\frac{\pi}{6} = \frac{1}{30} t$ (A1) por enfoque correcto

$t = 5\pi$ s A1

[3]

(c) $\frac{dv}{dt} = \frac{v^2 + 64}{240}$

$\frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{v^2 + 64}{240}$ A1

$v\frac{dv}{ds} = \frac{v^2 + 64}{240}$ A1

$\frac{240v}{v^2 + 64} dv = ds$ M1

$\int \frac{240v}{v^2 + 64} dv = \int ds$ A1

$s = \int \frac{240v}{v^2 + 64} dv$ AG

[4]

(d) $s = \int \frac{240v}{v^2 + 64} dv$

Sea $u = v^2 + 64$. (M1) por sustitución

$\frac{du}{dv} = 2v \Rightarrow 240v dv = 120 du$

$\therefore s = \int \frac{1}{u} \cdot 120 du$ A1

$s = 120 \ln|u| + D$

$s = 120 \ln(v^2 + 64) + D$ A1

$0 = 120 \ln(0^2 + 64) + D$ (M1) por sustitución

$D = -120 \ln 64$ (A1) por valor correcto

$\therefore s = 120 \ln(v^2 + 64) - 120 \ln 64$

$s = 120 \ln\left(\left(\frac{8}{3}\sqrt{3}\right)^2 + 64\right) - 120 \ln 64$

$s = 34,52184869 \text{ m}$

$s = 34,5 \text{ m}$ A1

[6]

12. (a) $\left(\cos\frac{\theta}{7}+\mathrm{i}\,\mathrm{sen}\frac{\theta}{7}\right)^7$

$=\cos^7\frac{\theta}{7}+\binom{7}{1}\mathrm{i}\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}+\binom{7}{2}\mathrm{i}^2\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$+\binom{7}{3}\mathrm{i}^3\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}+\binom{7}{4}\mathrm{i}^4\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}$ A2

$+\binom{7}{5}\mathrm{i}^5\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}+\binom{7}{6}\mathrm{i}^6\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}$

$+\mathrm{i}^7\mathrm{sen}^7\frac{\theta}{7}$

$=\cos^7\frac{\theta}{7}+7\mathrm{i}\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$-35\mathrm{i}\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}$ A1

$+21\mathrm{i}\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}-\mathrm{i}\,\mathrm{sen}^7\frac{\theta}{7}$

$\therefore \cos\theta+\mathrm{i}\,\mathrm{sen}\,\theta$

$=\cos^7\frac{\theta}{7}+7\mathrm{i}\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$-35\mathrm{i}\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}$ M1

$+21\mathrm{i}\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}-\mathrm{i}\,\mathrm{sen}^7\frac{\theta}{7}$

$=\cos^7\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}$

$+\mathrm{i}\begin{pmatrix}7\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-35\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}\\ +21\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-\mathrm{sen}^7\frac{\theta}{7}\end{pmatrix}$

$\therefore \cos\theta=\cos^7\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\mathrm{sen}^2\frac{\theta}{7}$

$+35\cos^3\frac{\theta}{7}\mathrm{sen}^4\frac{\theta}{7}-7\cos\frac{\theta}{7}\mathrm{sen}^6\frac{\theta}{7}$ y

$\mathrm{sen}\,\theta=7\cos^6\frac{\theta}{7}\mathrm{sen}\frac{\theta}{7}-35\cos^4\frac{\theta}{7}\mathrm{sen}^3\frac{\theta}{7}$

$+21\cos^2\frac{\theta}{7}\mathrm{sen}^5\frac{\theta}{7}-\mathrm{sen}^7\frac{\theta}{7}$. A2

[6]

(b) $\tan\theta$

$$=\frac{\operatorname{sen}\theta}{\cos\theta}$$

$$=\frac{7\cos^6\frac{\theta}{7}\operatorname{sen}\frac{\theta}{7}-35\cos^4\frac{\theta}{7}\operatorname{sen}^3\frac{\theta}{7}+21\cos^2\frac{\theta}{7}\operatorname{sen}^5\frac{\theta}{7}-\operatorname{sen}^7\frac{\theta}{7}}{\cos^7\frac{\theta}{7}-21\cos^5\frac{\theta}{7}\operatorname{sen}^2\frac{\theta}{7}+35\cos^3\frac{\theta}{7}\operatorname{sen}^4\frac{\theta}{7}-7\cos\frac{\theta}{7}\operatorname{sen}^6\frac{\theta}{7}}$$ M1A1

$$=\frac{7\tan\frac{\theta}{7}-35\tan^3\frac{\theta}{7}+21\tan^5\frac{\theta}{7}-\tan^7\frac{\theta}{7}}{1-21\tan^2\frac{\theta}{7}+35\tan^4\frac{\theta}{7}-7\tan^6\frac{\theta}{7}}$$ A1

Sea $x=\tan\frac{\theta}{7}$.

$$\tan\theta=\frac{7x-35x^3+21x^5-x^7}{1-21x^2+35x^4-7x^6}$$ M1

$x^6-21x^4+35x^2-7=0$

$$\frac{-x(x^6-21x^4+35x^2-7)}{1-21x^2+35x^4-7x^6}=0$$

$$\frac{7x-35x^3+21x^5-x^7}{1-21x^2+35x^4-7x^6}=0$$

$\tan\theta=0$ M1

$\theta=0$, π, 2π, 3π, 4π, 5π o 6π

$\therefore x=\tan\frac{0}{7}$, $x=\tan\frac{\pi}{7}$, $x=\tan\frac{2\pi}{7}$, $x=\tan\frac{3\pi}{7}$,

$x=\tan\frac{4\pi}{7}$, $x=\tan\frac{5\pi}{7}$ o $x=\tan\frac{6\pi}{7}$ A1

$x=0$ (*Rechazada*), $x=\tan\frac{\pi}{7}$, $x=\tan\frac{2\pi}{7}$,

$x=\tan\frac{3\pi}{7}$, $x=\tan\frac{4\pi}{7}$, $x=\tan\frac{5\pi}{7}$ o $x=\tan\frac{6\pi}{7}$ A1

Por tanto, la ecuación $x^6-21x^4+35x^2-7=0$ tiene seis raíces. AG

[7]

(c) (i) $\sum_{r=1}^{7} \tan\frac{r\pi}{7}$

$= \sum_{r=1}^{6} \tan\frac{r\pi}{7} + \tan\frac{7\pi}{7}$ M1

$= -\frac{0}{1} + 0$ A1

$= 0$ A1

(ii) $\left(\tan\frac{\pi}{7}\right)\left(\tan\frac{2\pi}{7}\right)\left(\tan\frac{3\pi}{7}\right)\left(\tan\frac{4\pi}{7}\right)\left(\tan\frac{5\pi}{7}\right)\left(\tan\frac{6\pi}{7}\right) = -7$ M1A1

$\left(\tan\frac{\pi}{7}\right)\left(\tan\frac{2\pi}{7}\right)\left(\tan\frac{3\pi}{7}\right)\left(\tan\left(\pi - \frac{3\pi}{7}\right)\right)\left(\tan\left(\pi - \frac{2\pi}{7}\right)\right)\left(\tan\left(\pi - \frac{\pi}{7}\right)\right) = -7$

$\left(\tan\frac{\pi}{7}\right)\left(\tan\frac{2\pi}{7}\right)\left(\tan\frac{3\pi}{7}\right)\left(-\tan\frac{3\pi}{7}\right)\left(-\tan\frac{2\pi}{7}\right)\left(-\tan\frac{\pi}{7}\right) = -7$ A1

$\left(\tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{3\pi}{7}\right)^2 = 7$

$\therefore \tan\frac{\pi}{7}\tan\frac{2\pi}{7}\tan\frac{3\pi}{7} = \sqrt{7}$ A1

[7]

Solución de Práctica de Prueba 3 de AE NS Set 3

1.	(a)		$r_1 + r_2 = -\frac{a_1}{1}$	(A1) por sustitución
			$a_1 = -r_1 - r_2$	A1
			$r_1 r_2 = \frac{a_0}{1}$	(A1) por sustitución
			$a_0 = r_1 r_2$	A1

[4]

(b)	(i)	a_1	
		$= -r_1 - r_2$	
		$= -(r_1 + r_2)$	
		$= -S_1$	A1
	(ii)	$\frac{S_1^{\,2} - S_2}{2}$	
		$= \frac{(r_1 + r_2)^2 - (r_1^2 + r_2^2)}{2}$	
		$= \frac{r_1^2 + 2r_1 r_2 + r_2^2 - r_1^2 - r_2^2}{2}$	M1A1
		$= \frac{2r_1 r_2}{2}$	
		$= r_1 r_2$	A1
		$= a_0$	
		$\therefore a_0 = \frac{S_1^{\,2} - S_2}{2}$	AG

[4]

(c)	(i)	$a_2 = -S_1$	A1

(ii) $\dfrac{S_1^{\,2} - S_2}{2}$

$= \dfrac{(r_1 + r_2 + r_3)^2 - (r_1^{\,2} + r_2^{\,2} + r_3^{\,2})}{2}$

$= \dfrac{r_1^{\,2} + r_1r_2 + r_1r_3 + r_1r_2 + r_2^{\,2} + r_2r_3 + r_1r_3 + r_2r_3 + r_3^{\,2} - r_1^{\,2} - r_2^{\,2} - r_3^{\,2}}{2}$ M1A1

$= \dfrac{2r_1r_2 + 2r_1r_3 + 2r_2r_3}{2}$

$= r_1r_2 + r_1r_3 + r_2r_3$ R1

$= a_1$

$\therefore a_1 = \dfrac{S_1^{\,2} - S_2}{2}$ es cierto. A1

[5]

(d) $ka_0 = S_1^{\,3} - 3S_1S_2 + 2S_3$

$k(-r_1r_2r_3) = (r_1 + r_2 + r_3)^3$ (A1) por enfoque correcto

$-3(r_1 + r_2 + r_3)(r_1^{\,2} + r_2^{\,2} + r_3^{\,2}) + 2(r_1^{\,3} + r_2^{\,3} + r_3^{\,3})$

$-kr_1r_2r_3 = (r_1 + r_2 + r_3)(r_1 + r_2 + r_3)^2$

$-3(r_1 + r_2 + r_3)(r_1^{\,2} + r_2^{\,2} + r_3^{\,2}) + 2(r_1^{\,3} + r_2^{\,3} + r_3^{\,3})$

$-kr_1r_2r_3$

$= (r_1 + r_2 + r_3)(r_1^{\,2} + r_2^{\,2} + r_3^{\,2} + 2r_1r_2 + 2r_1r_3 + 2r_2r_3)$ M1A1

$-3(r_1 + r_2 + r_3)(r_1^{\,2} + r_2^{\,2} + r_3^{\,2}) + 2(r_1^{\,3} + r_2^{\,3} + r_3^{\,3})$

El coeficiente de $r_1r_2r_3$ en el lado derecho

$= 2 + 2 + 2$ A1

$= 6$

$\therefore k = -6$ A1

[5]

(e) $a_{n-1} = -S_1$ A1

$a_{n-2} = \dfrac{S_1^{\,2} - S_2}{2}$ A1

$a_{n-3} = -\dfrac{1}{6}S_1^{\,3} + \dfrac{1}{2}S_1S_2 - \dfrac{1}{3}S_3$ A1

[3]

(f) $\begin{cases} u+v+w=14 \\ u^2+v^2+w^2=86 \\ u^3+v^3+w^3=560 \end{cases}$

Sean $S_1=14$, $S_2=86$ y $S_3=560$. (M1) por enfoque válido

u, v y w son las raíces de la ecuación

$x^3+a_2x^2+a_1x+a_0=0$, donde $a_2=-S_1$,

$a_1=\frac{S_1{}^2-S_2}{2}$ y $a_0=-\frac{1}{6}S_1{}^3+\frac{1}{2}S_1S_2-\frac{1}{3}S_3$. A1

$a_2=-14$ A1

$a_1=\frac{14^2-86}{2}$

$a_1=55$ A1

$a_0=-\frac{1}{6}(14)^3+\frac{1}{2}(14)(86)-\frac{1}{3}(560)$

$a_0=-42$ A1

Por lo tanto, u, v y w son las raíces de la ecuación $x^3-14x^2+55x-42=0$. R1

Considerando la gráfica de

$y=x^3-14x^2+55x-42$, $x=1$, $x=6$ o $x=7$.

$\therefore u=1$, $v=6$, $w=7$ A3

[9]

2. (a) $\cos(A+B)x+\cos(A-B)x$

$\cos Ax\cos Bx-\operatorname{sen}Ax\operatorname{sen}Bx$

$+\cos Ax\cos Bx+\operatorname{sen}Ax\operatorname{sen}Bx$ A2

$=2\cos Ax\cos Bx$ AG

[2]

(b) $\int_0^{\pi}\cos Ax\cos Bx\,\mathrm{d}x$

$=\frac{1}{2}\int_0^{\pi}(\cos(A+B)x+\cos(A-B)x)\mathrm{d}x$ (A1) por sustitución

$=\frac{1}{2}\left[\frac{1}{A+B}\operatorname{sen}(A+B)x+\frac{1}{A-B}\operatorname{sen}(A-B)x\right]_0^{\pi}$ A1

$=\frac{1}{2}\left[\left(\frac{1}{A+B}\operatorname{sen}(A+B)\pi+\frac{1}{A-B}\operatorname{sen}(A-B)\pi\right)-\left(\frac{1}{A+B}\operatorname{sen}0+\frac{1}{A-B}\operatorname{sen}0\right)\right]$ M1

$=0$ A1

[4]

(c) (i) $\frac{1}{z}$

$=\frac{1}{\cos\theta+\mathrm{i}\operatorname{sen}\theta}$

$=\frac{\cos\theta-\mathrm{i}\operatorname{sen}\theta}{(\cos\theta+\mathrm{i}\operatorname{sen}\theta)(\cos\theta-\mathrm{i}\operatorname{sen}\theta)}$ M1

$=\frac{\cos\theta-\mathrm{i}\operatorname{sen}\theta}{\cos^2\theta-\mathrm{i}^2\operatorname{sen}^2\theta}$ A1

$=\frac{\cos\theta-\mathrm{i}\operatorname{sen}\theta}{\cos^2\theta+\operatorname{sen}^2\theta}$

$=\cos\theta-\mathrm{i}\operatorname{sen}\theta$ A1

(ii) $z+\frac{1}{z}=(\cos\theta+\mathrm{i}\operatorname{sen}\theta)+(\cos\theta-\mathrm{i}\operatorname{sen}\theta)$ (M1) por enfoque válido

$z+\frac{1}{z}=2\cos\theta$

$\cos\theta=\frac{1}{2}\left(z+\frac{1}{z}\right)$ A1

[5]

(d) $\cos^3\theta$

$$=\left(\frac{1}{2}\right)^3\left(z+\frac{1}{z}\right)^3$$

$$=\frac{1}{8}\left(z^3+\binom{3}{1}z^2\cdot\frac{1}{z}+\binom{3}{2}z\cdot\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3\right)$$ M1A1

$$=\frac{1}{8}\begin{pmatrix}\cos3\theta+\mathrm{i}\operatorname{sen}3\theta+3\cos\theta+3\mathrm{i}\operatorname{sen}\theta\\+\dfrac{3}{\cos\theta+\mathrm{i}\operatorname{sen}\theta}+\dfrac{1}{\cos3\theta+\mathrm{i}\operatorname{sen}3\theta}\end{pmatrix}$$ A1

$$=\frac{1}{8}\begin{pmatrix}\cos3\theta+\mathrm{i}\operatorname{sen}3\theta+3\cos\theta+3\mathrm{i}\operatorname{sen}\theta\\+3(\cos\theta-\mathrm{i}\operatorname{sen}\theta)+\cos3\theta-\mathrm{i}\operatorname{sen}3\theta\end{pmatrix}$$ A1

$$=\frac{1}{8}(2\cos3\theta+6\cos\theta)$$

$$=\frac{1}{4}(\cos3\theta+3\cos\theta)$$ AG

[4]

(e) $\cos^n\theta$

$$=\left(\frac{1}{2}\right)^n\left(z+\frac{1}{z}\right)^n$$

$$=\frac{1}{2^n}\begin{pmatrix}z^n+\binom{n}{1}z^{n-1}\cdot\dfrac{1}{z}+\binom{n}{2}z^{n-2}\cdot\left(\dfrac{1}{z}\right)^2\\+\cdots+\binom{n}{n-1}z\cdot\left(\dfrac{1}{z}\right)^{n-1}+\left(\dfrac{1}{z}\right)^n\end{pmatrix}$$ M1A1

$$=\frac{1}{2^n}\begin{pmatrix}z^n+\binom{n}{1}z^{n-2}+\binom{n}{2}z^{n-4}\\+\cdots+\binom{n}{n-1}\dfrac{1}{z^{n-2}}+\dfrac{1}{z^n}\end{pmatrix}$$ M1

$$=\frac{1}{2^n}\sum_{r=0}^{n}\binom{n}{r}\cos(n-2r)\theta$$ A2

[5]

(f) $\int_0^{\pi} \cos 6x \cos^5 x \mathrm{d}x$

$$= \int_0^{\pi} \cos 6x \left(\frac{1}{2^5} \sum_{r=0}^{5} \binom{5}{r} \cos(5-2r)x \right) \mathrm{d}x$$ (A1) por sustitución

$$= \frac{1}{32} \int_0^{\pi} \cos 6x \begin{pmatrix} \cos 5x + 5\cos 3x + 10\cos x \\ +10\cos(-x) + 5\cos(-3x) + \cos(-5x) \end{pmatrix} \mathrm{d}x$$ M1

$$= \frac{1}{32} \int_0^{\pi} \cos 6x \begin{pmatrix} \cos 5x + 5\cos 3x + 10\cos x \\ +10\cos x + 5\cos 3x + \cos 5x \end{pmatrix} \mathrm{d}x$$ A1

$$= \frac{1}{32} \int_0^{\pi} \cos 6x (2\cos 5x + 10\cos 3x + 20\cos x) \mathrm{d}x$$

$$= \frac{1}{32} \int_0^{\pi} 2\cos 6x \cos 5x \mathrm{d}x + \frac{1}{32} \int_0^{\pi} 10\cos 6x \cos 3x \mathrm{d}x$$

$$+ \frac{1}{32} \int_0^{\pi} 20\cos 6x \cos x \mathrm{d}x$$

$$= \frac{1}{16} \int_0^{\pi} \cos 6x \cos 5x \mathrm{d}x + \frac{5}{16} \int_0^{\pi} \cos 6x \cos 3x \mathrm{d}x$$

$$+ \frac{5}{8} \int_0^{\pi} \cos 6x \cos x \mathrm{d}x$$ A1

$$= \frac{1}{16}(0) + \frac{5}{16}(0) + \frac{5}{8}(0)$$

$$= 0$$ A1

[5]

Solución de Práctica de Prueba 1 de AE NS Set 4

Sección A

1. (a) El área de la region sombreada

$=\frac{1}{2}(20)^2(1,5)$ (A1) por sustitución

$=300\text{ cm}^2$ A1

[2]

(b) La longitud del arco ABC

$=(20)(1,5)$ (A1) por sustitución

$=30\text{ cm}$ A1

[2]

(c) El perimetro requerido

$=2\pi(20)-30+20+20$ (M1) por enfoque válido

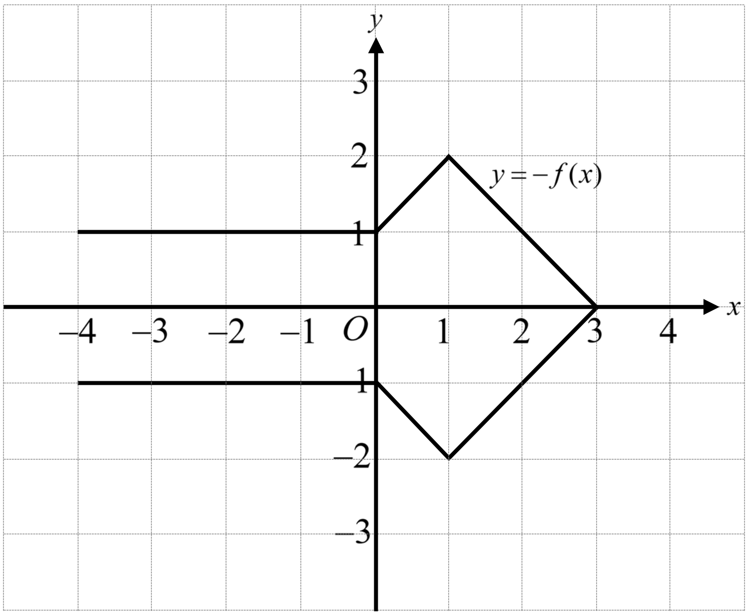
$=(40\pi+10)\text{ cm}$ A1

[2]

2. (a) Por los puntos de intersección con el eje x e y correctos A1

Por dos puntos correctos $(-4, 1)$ y $(1, 2)$ A1

[2]



(b) $p = 2$ A2

$q = -1$ A2

[4]

3. (a) $\log_4 64$

$= \log_4 4^3$ (A1) por enfoque correcto

$= 3$ A1

[2]

(b) $\log_{12} 36 + \log_{12} 4$

$= \log_{12} 144$ (A1) por enfoque correcto

$= \log_{12} 12^2$

$= 2$ A1

[2]

(c) $\log_2 11 - \log_2 88$

$= \log_2 \dfrac{1}{8}$ (A1) por enfoque correcto

$= \log_2 2^{-3}$

$= -3$ A1

[2]

4. (a) $a = 2(-\text{sen}\,\pi t)(\pi) + 0$ (A1) por derivadas correctas

$a = -2\pi\,\text{sen}\,\pi t$ A1

[2]

(b) $s = \int (2\cos \pi t + \pi)\mathrm{d}t$ (M1) por integral indefinida

$s = \int 2\cos \pi t \mathrm{d}t + \int \pi \mathrm{d}t$

Sea $u = \pi t$ $\frac{\mathrm{d}u}{\mathrm{d}t} = \pi \Rightarrow \frac{1}{\pi}\mathrm{d}u = \mathrm{d}t$

(A1) por sustitución

$s = \int \frac{2}{\pi}\cos u \mathrm{d}u + \int \pi \mathrm{d}t$

$s = \frac{2}{\pi}\text{sen}\,u + \pi t + C$ A1

$s = \frac{2}{\pi}\text{sen}\,\pi t + \pi t + C$

$\therefore -3 = \frac{2}{\pi}\text{sen}\,0 + 0 + C$ (M1) por sustitución

$C = -3$

$\therefore s = \frac{2}{\pi}\text{sen}\,\pi t + \pi t - 3$ A1

[5]

5. (a) $1 < D < 5$ A1

[1]

(b) 6 horas A1

[1]

(c) (i) La media requerida

$= 10,5 + 1,5$ (M1) por enfoque válido

$= 12$ A1

(ii) La varianza requerida

$= 2^2$ (M1)(A1) por enfoque correcto

$= 4$ A1

[5]

6. $\lim_{x\to 0}\dfrac{1+3x-\cos\frac{\pi}{3}x}{\ln(1+x)}$

$=\lim_{x\to 0}\dfrac{0+3-\left(-\operatorname{sen}\frac{\pi}{3}x\right)\left(\frac{\pi}{3}\right)}{\left(\frac{1}{x+1}\right)(1)}\left(\because\frac{0}{0}\right)$ M1A2

$=\lim_{x\to 0}(x+1)\left(3+\frac{\pi}{3}\operatorname{sen}\frac{\pi}{3}x\right)$

$=(0+1)\left(3+\frac{\pi}{3}\operatorname{sen}\frac{\pi}{3}(0)\right)$ M1

$=3$ A1

[5]

7. $\tan x+\cot x+\dfrac{4\sqrt{3}}{3}=0$

$\tan x+\cot x=-\dfrac{4\sqrt{3}}{3}$

$\dfrac{\operatorname{sen}x}{\cos x}+\dfrac{\cos x}{\operatorname{sen}x}=-\dfrac{4\sqrt{3}}{3}$ (A1) por sustitución

$\dfrac{\operatorname{sen}^2 x+\cos^2 x}{\operatorname{sen}x\cos x}=-\dfrac{4\sqrt{3}}{3}$

$1=-\dfrac{4\sqrt{3}}{3}\operatorname{sen}x\cos x$ (M1) por enfoque válido

$-\sqrt{3}=2(2\operatorname{sen}x\cos x)$

$\operatorname{sen}2x=-\dfrac{\sqrt{3}}{2}$ A1

$2x=\pi+\dfrac{\pi}{3}$ o $2x=2\pi-\dfrac{\pi}{3}$

$x=\dfrac{2\pi}{3}$ o $x=\dfrac{5\pi}{6}$ A2

[5]

8. (a) $L_1 : \begin{cases} x = 17 + 5t \\ y = 1 - 2t \\ z = 10 + 3t \end{cases}$

$(17 + 5t) - 8 = 3 - (10 + 3t)$ (M1) por ecuación

$9 + 5t = -7 - 3t$

$16 = -8t$

$t = -2$ A1

$\therefore \begin{cases} x = 17 + 5(-2) = 7 \\ y = 1 - 2(-2) = 5 \\ z = 10 + 3(-2) = 4 \end{cases}$ (M1) por sustitución

Por tanto, las coordenadas de P son $(7, 5, 4)$. A1

[4]

(b) $\overrightarrow{\mathrm{RQ}} = -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

$\therefore \overrightarrow{\mathrm{OQ}} - \overrightarrow{\mathrm{OR}} = -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$ (M1) por enfoque válido

$((17 + 5t)\mathbf{i} + (1 - 2t)\mathbf{j} + (10 + 3t)\mathbf{k}) - (3\mathbf{i} + 5\mathbf{k})$

$= -\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

$17 + 5t - 3 = -1$

$5t = -15$

$t = -3$ (A1) por valor correcto

$\therefore \begin{cases} x = 17 + 5(-3) = 2 \\ y = 1 - 2(-3) = 7 \\ z = 10 + 3(-3) = 1 \end{cases}$ (M1) por sustitución

Por tanto, las coordenadas de Q son $(2, 7, 1)$. A1

[4]

9. Cuando $n=1$,

$5-21(1)+4^1=-12$

$5-21(1)+4^1=3(-4)$ A1

Por lo tanto, el enunciado es cierto cuando $n=1$.

Suponemos que es cierto para $n=k$. M1

$5-21k+4^k=3M$, donde $M\in\mathbb{Z}$.

Cuando $n=k+1$,

$5-21(k+1)+4^{k+1}$

$=5-21k-21+4(4^k)$ M1

$=-16-21k+4(3M+21k-5)$ A1

$=-16-21k+12M+84k-20$

$=12M+63k-36$ M1

$=3(4M+21k-12)$, donde $4M+21k-12\in\mathbb{Z}$. A1

Por lo tanto, el enunciado es cierto cuando $n=k+1$.

Por lo tanto, el enunciado es cierto para todo $n\in\mathbb{Z}^+$. R1

[7]

Sección B

10. (a) (i) La probabilidad requerida

$= \frac{3}{n}$ A1

(ii) La probabilidad requerida

$= \left(\frac{n-3}{n}\right)\left(\frac{n-4}{n-1}\right)\left(\frac{3}{n-2}\right)$ (A1) por enfoque correcto

$= \frac{3(n-3)(n-4)}{n(n-1)(n-2)}$ A1

[3]

(b) La probabilidad requerida

$= \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$ (A1) por enfoque correcto

$= \frac{1}{8}$ A1

[2]

(c) El juego es justo si la ganancia esperada es cero, lo que equivale a que la cantidad esperada de dinero recuperado sea $10\$$. R1

$\therefore \left(\frac{3}{10}\right)(10) + \left(\left(\frac{7}{10}\right)\left(\frac{3}{9}\right)\right)(10)$

$+ \left(\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right)\right)(25x) + \left(\frac{1}{8}\right)(21x)$ M1A2

$+ \left(1 - \frac{3}{10} - \left(\frac{7}{10}\right)\left(\frac{3}{9}\right) - \left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{3}{8}\right) - \frac{1}{8}\right)(0) = 10$

$3 + \frac{7}{3} + \frac{35}{8}x + \frac{21}{8}x = 10$ M1A1

$7x = \frac{14}{3}$ A1

$x = \frac{2}{3}$ AG

[7]

11. (a) $z^{20} = 1$

$z^{20} = \cos 0 + \mathrm{i}\,\mathrm{sen}\, 0$ A1

$z = \cos\left(\dfrac{0+2k\pi}{20}\right) + \mathrm{i}\,\mathrm{sen}\left(\dfrac{0+2k\pi}{20}\right)$ M1

$(k = 0, 1, 2, \cdots, 18, 19)$

$z = \cos 0 + \mathrm{i}\,\mathrm{sen}\, 0$, $z = \cos\dfrac{\pi}{10} + \mathrm{i}\,\mathrm{sen}\dfrac{\pi}{10}$,

$z = \cos\dfrac{\pi}{5} + \mathrm{i}\,\mathrm{sen}\dfrac{\pi}{5}$, …,

$z = \cos\dfrac{9\pi}{5} + \mathrm{i}\,\mathrm{sen}\dfrac{9\pi}{5}$ o $z = \cos\dfrac{19\pi}{10} + \mathrm{i}\,\mathrm{sen}\dfrac{19\pi}{10}$ (A1) por valores correctos

$-\dfrac{\pi}{2} \le \arg(z) \le 0$

$\therefore z = \mathrm{cis}0$, $z = \mathrm{cis}\left(-\dfrac{\pi}{2}\right)$, $z = \mathrm{cis}\left(-\dfrac{2\pi}{5}\right)$,

$z = \mathrm{cis}\left(-\dfrac{3\pi}{10}\right)$, $z = \mathrm{cis}\left(-\dfrac{\pi}{5}\right)$ o $z = \mathrm{cis}\left(-\dfrac{\pi}{10}\right)$ A3

[6]

(b) $1 + \cos\left(-\dfrac{2\pi}{5}\right) + \cos\left(-\dfrac{3\pi}{10}\right)$
$+ \cos\left(-\dfrac{\pi}{5}\right) + \cos\left(-\dfrac{\pi}{10}\right)$ A1

[1]

(c) $\operatorname{Im} S$

$= -1 + \operatorname{sen}\left(-\frac{2\pi}{5}\right) + \operatorname{sen}\left(-\frac{3\pi}{10}\right)$

$+ \operatorname{sen}\left(-\frac{\pi}{5}\right) + \operatorname{sen}\left(-\frac{\pi}{10}\right)$ A1

$= -1 - \operatorname{sen}\frac{2\pi}{5} - \operatorname{sen}\frac{3\pi}{10} - \operatorname{sen}\frac{\pi}{5} - \operatorname{sen}\frac{\pi}{10}$ M1

$= -1 - \cos\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) - \cos\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$

$- \cos\left(\frac{\pi}{2} - \frac{\pi}{5}\right) - \cos\left(\frac{\pi}{2} - \frac{\pi}{10}\right)$ A1

$= -1 - \cos\frac{\pi}{10} - \cos\frac{\pi}{5} - \cos\frac{3\pi}{10} - \cos\frac{4\pi}{10}$ M1

$= -\left(\begin{array}{l} 1 + \cos\left(-\frac{2\pi}{5}\right) + \cos\left(-\frac{3\pi}{10}\right) \\ + \cos\left(-\frac{\pi}{5}\right) + \cos\left(-\frac{\pi}{10}\right) \end{array}\right)$ A1

$= -\operatorname{Re} S$

$\therefore \frac{\operatorname{Re} S}{\operatorname{Im} S} = -1$ AG

[5]

(d) (i) $\cos\left(-\frac{\pi}{5}\right)$

$= \cos\frac{\pi}{5}$

$= 2\cos^2\frac{\pi}{10} - 1$ (A1) por sustitución

$= 2\left(\frac{\sqrt{10 + 2\sqrt{5}}}{4}\right)^2 - 1$

$= \frac{10 + 2\sqrt{5}}{8} - 1$ (M1) por enfoque válido

$= \frac{10 + 2\sqrt{5} - 8}{8}$

$= \frac{1 + \sqrt{5}}{4}$ A1

(ii) $\cos\left(-\frac{2\pi}{5}\right)$

$= 2\cos^2\left(-\frac{\pi}{5}\right)-1$ (A1) por sustitución

$= 2\left(\frac{1+\sqrt{5}}{4}\right)^2-1$

$= \frac{1+2\sqrt{5}+5}{8}-1$ (M1) por enfoque válido

$= \frac{6+2\sqrt{5}-8}{8}$

$= \frac{\sqrt{5}-1}{4}$ A1

[6]

(e) $\operatorname{Im} S$

$= -\operatorname{Re} S$

$= -\begin{pmatrix} 1+\cos\left(-\frac{2\pi}{5}\right)+\cos\left(-\frac{3\pi}{10}\right) \\ +\cos\left(-\frac{\pi}{5}\right)+\cos\left(-\frac{\pi}{10}\right) \end{pmatrix}$ M1

$= -\left(1+\frac{\sqrt{5}-1}{4}+\frac{\sqrt{10-2\sqrt{5}}}{4}+\frac{1+\sqrt{5}}{4}+\frac{\sqrt{10+2\sqrt{5}}}{4}\right)$ A1

$= -\left(1+\frac{\sqrt{5}}{2}+\frac{\sqrt{10-2\sqrt{5}}+\sqrt{10+2\sqrt{5}}}{4}\right)$

$= -\left(\frac{4}{4}+\frac{2\sqrt{5}}{4}+\frac{\sqrt{10-2\sqrt{5}}+\sqrt{10+2\sqrt{5}}}{4}\right)$

$= -\frac{4+2\sqrt{5}+\sqrt{10-2\sqrt{5}}+\sqrt{10+2\sqrt{5}}}{4}$ AG

[2]

12. (a) (i) $f(x) = g(x)$

$\therefore \operatorname{sen} 2\pi y = -\operatorname{sen} \pi y$ M1

$2 \operatorname{sen} \pi y \cos \pi y + \operatorname{sen} \pi y = 0$ A1

$\operatorname{sen} \pi y (2\cos \pi y + 1) = 0$ A1

$\operatorname{sen} \pi y = 0$ o $\cos \pi y = -\dfrac{1}{2}$

$\pi y = 0$ o $\pi y = \dfrac{2\pi}{3}$ A1

$y = 0$ (*Rechazada*) o $y = \dfrac{2}{3}$

$\therefore r = \dfrac{2}{3}$ AG

(ii) El área de la región

$= \int_{\frac{2}{3}}^{1} (g(y) - f(y))\,\mathrm{d}y$ A1

$= \int_{\frac{2}{3}}^{1} (-\operatorname{sen} \pi y - \operatorname{sen} 2\pi y)\,\mathrm{d}y$

$= \left[\dfrac{1}{\pi}\cos \pi y + \dfrac{1}{2\pi}\cos 2\pi y\right]_{\frac{2}{3}}^{1}$ A1

$= \left(\dfrac{1}{\pi}\cos \pi(1) + \dfrac{1}{2\pi}\cos 2\pi(1)\right)$
$-\left(\dfrac{1}{\pi}\cos \pi\left(\dfrac{2}{3}\right) + \dfrac{1}{2\pi}\cos 2\pi\left(\dfrac{2}{3}\right)\right)$ M1

$= \left(-\dfrac{1}{\pi} + \dfrac{1}{2\pi}\right) - \left(\dfrac{1}{\pi}\left(-\dfrac{1}{2}\right) + \dfrac{1}{2\pi}\left(-\dfrac{1}{2}\right)\right)$ A1

$= -\dfrac{1}{2\pi} - \left(-\dfrac{1}{2\pi} - \dfrac{1}{4\pi}\right)$ M1

$= -\dfrac{1}{2\pi} + \dfrac{1}{2\pi} + \dfrac{1}{4\pi}$

$= \dfrac{1}{4\pi}$ AG

[9]

(b) $a\,\text{sen}\,2\pi\left(\frac{3}{4}\right)=-\frac{\sqrt{2}}{2}$ (M1) por sustitución

$-a=-\frac{\sqrt{2}}{2}$ A1

$a=\frac{\sqrt{2}}{2}$ A1

[3]

(c) $f(x)=g(x)$

$\therefore a\,\text{sen}\,2\pi y=-\text{sen}\,\pi y$ M1

$2a\,\text{sen}\,\pi y\cos\pi y+\text{sen}\,\pi y=0$ A1

$\text{sen}\,\pi y(2a\cos\pi y+1)=0$ A1

$2a\cos\pi y+1=0$ M1

$2a\cos\pi y=-1$

$\cos\pi y=-\frac{1}{2a}$ A1

$\therefore \text{sen}\,\pi y$

$=\sqrt{1-\cos^2\pi y}$ A1

$=\sqrt{1-\left(-\frac{1}{2a}\right)^2}$

$=\sqrt{1-\frac{1}{4a^2}}$ M1

$=\sqrt{\frac{4a^2-1}{4a^2}}$ A1

$\pi y=\text{arcsen}\sqrt{\frac{4a^2-1}{4a^2}}$ M1

$\therefore r=\frac{1}{\pi}\text{arcsen}\sqrt{\frac{4a^2-1}{4a^2}}$ AG

[9]

Solución de Práctica de Prueba 2 de AE NS Set 4

Sección A

1. (a) (i) $(3, -127)$ A2

(ii) $f(x) = 3(x-3)^2 - 127$ A2

[4]

(b) $3x^2 - 18x - 100 = -52$

$3x^2 - 18x - 48 = 0$ (A1) por ecuación correcta

$3(x+2)(x-8) = 0$

$x = -2$ o $x = 8$ A2

[3]

2. (a) p es negativo ya que el primer punto crítico es un mínimo. R1

$p = -\dfrac{4,3}{2}$ A1

$p = -2,15$ AG

[2]

(c) (i) El período

$= 13,75 - 2,75$ (M1) por enfoque válido

$= 11$ horas (A1) por valor correcto

$\therefore q = \dfrac{2\pi}{11}$ A1

(ii) $r = \dfrac{(1,9 + 4,3) + 1,9}{2}$ (M1) por enfoque válido

$r = 4,05$ A1

[5]

3. (a) $\text{B}\hat{\text{A}}\text{C}$

$= \pi - 0,88 - 1,23$ (M1) por enfoque válido

$= 1,031592654$ A1

$\frac{\text{AB}}{\text{sen A}\hat{\text{C}}\text{B}} = \frac{\text{BC}}{\text{sen B}\hat{\text{A}}\text{C}}$ (M1) por regla del seno

$\frac{\text{AB}}{\text{sen}\,1,23} = \frac{20}{\text{sen}\,1,031592654}$ (A1) por sustitución

$\text{AB} = 21,96641928 \text{ cm}$

$\text{AB} = 22,0 \text{ cm}$ A1

[5]

(b) $\text{AB}^2 = \text{OA}^2 + \text{OB}^2 - 2(\text{OA})(\text{OB})\cos \text{A}\hat{\text{O}}\text{B}$ M1

$\text{AB}^2 = r^2 + r^2 - 2(r)(r)\cos \text{A}\hat{\text{O}}\text{B}$ A1

$\text{AB}^2 = 2r^2 - 2r^2 \cos \text{A}\hat{\text{O}}\text{B}$

$\text{AB}^2 = 2r^2(1 - \cos \text{A}\hat{\text{O}}\text{B})$ A1

$r^2 = \frac{\text{AB}^2}{2(1 - \cos \text{A}\hat{\text{O}}\text{B})}$ AG

[3]

4. (a) La razón común r

$= \frac{3k^2 - 4k^3}{k^2}$ (M1) por enfoque válido

$= 3 - 4k$ A1

[2]

(b) S_∞ existe si $-1 < r < 1$. R1

$\therefore -1 < 3 - 4k < 1$ M1

$-1 < 4k - 3 < 1$

$2 < 4k < 4$ A1

$\frac{1}{2} < k < 1$ AG

[3]

5. El término general

$=\binom{9}{r}\left(\frac{x}{h^2}\right)^{9-r}\left(-\frac{h}{x^2}\right)^r$ (M1) por expansión válida

$=\binom{9}{r}(-1)^r h^{3r-18}x^{9-3r}$

$9-3r=0$ (A1) por ecuación correcta

$3r=9$

$r=3$ (A1) por valor correcto

El término requerido

$=\binom{9}{3}(-1)^3 h^{3(3)-18}x^{9-3(3)}$

$=-\frac{84}{h^9}$ (A1) por término correcto

$-\frac{84}{h^9}=-\frac{21}{65536}$ (M1) por ecuación

$h^9=262144$

$h=4$ A1

[6]

6. (a) Sea $\frac{x^2+9}{(4-x)(5-2x)} \equiv A + \frac{B}{4-x} + \frac{C}{5-2x}$, donde A, B y C son constantes.

$$\frac{x^2+9}{(4-x)(5-2x)} \equiv \frac{A(4-x)(5-2x)}{(4-x)(5-2x)} + \frac{B(5-2x)}{(4-x)(5-2x)} + \frac{C(4-x)}{(4-x)(5-2x)}$$ M1

$$\frac{x^2+9}{(4-x)(5-2x)} \equiv \frac{20A-13Ax+2Ax^2+5B-2Bx+4C-Cx}{(4-x)(5-2x)}$$

$x^2+9 \equiv 2Ax^2+(-13A-2B-C)x+(20A+5B+4C)$ A1

$2A=1$

$A=\frac{1}{2}$ A1

$0=-13\left(\frac{1}{2}\right)-2B-C$

$C=-\frac{13}{2}-2B$

$9=20A+5B+4C$

$\therefore 9=20\left(\frac{1}{2}\right)+5B+4\left(-\frac{13}{2}-2B\right)$ A1

$9=10+5B-26-8B$

$25=-3B$

$B=-\frac{25}{3}$ A1

$\therefore C=-\frac{13}{2}-2\left(-\frac{25}{3}\right)$

$C=\frac{61}{6}$ A1

[6]

(b) $g(x)=-\frac{(4-x)(5-2x)}{x^2+9}$

El discriminante de x^2+9

$=0^2-4(1)(9)$ A1

$=-36<0$

Por lo tanto, el denominador es siempre distinto de cero.

Por tanto, $g(x)$ no tiene asíntota vertical. AG

[1]

7. (a) $\left\{x : -5 \le x \le \frac{2}{3}\right\}$ A2

[2]

(b) $f(x) = (3x-2)^2$

$y = (3x-2)^2$

$\Rightarrow x = (3y-2)^2$ (M1) por intercambiar variables

$-\sqrt{x} = 3y-2$

$-\sqrt{x}+2 = 3y$

$y = \frac{-\sqrt{x}+2}{3}$

$\therefore f^{-1}(x) = \frac{-\sqrt{x}+2}{3}$ A1

[2]

(c) $(f \circ g)(x) = x^4$

$g(x) = f^{-1}(x^4)$ M1

$g(x) = \frac{-\sqrt{x^4}+2}{3}$

$g(x) = \frac{-x^2+2}{3}$ A1

[2]

8. $\binom{12}{2} \times \binom{10}{r} \times \binom{10-r}{10-r} = 7920$ M1A1

$\binom{10}{r} = 120$ (A1) por simplificación

$\binom{10}{r} = \binom{10}{3}$ or $\binom{10}{r} = \binom{10}{7}$

$r = 3$ o $r = 7$ A2

[5]

9. (a) La desviación típica de X

$= \sqrt{E(X^2) - (E(X))^2}$ (M1) por enfoque válido

$= \sqrt{\int_{-4}^{0} x^2 \cdot \left(\frac{1}{20}x + \frac{1}{5}\right) dx + \int_{0}^{3} x^2 \cdot \left(-\frac{1}{15}x^2 + \frac{2}{15}x + \frac{1}{5}\right) dx - \left(\frac{13}{60}\right)^2}$ A1

$= \sqrt{2,279722222}$

$= 1,509874903$

$= 1,51$ A1

[3]

(b) $P(|X| > 2)$

$= P(X > 2 \text{ o } X < -2)$ (M1) por enfoque válido

$= P(X < -2) + P(X > 2)$

$= \int_{-4}^{-2} \left(\frac{1}{20}x + \frac{1}{5}\right) dx + \int_{2}^{3} \left(-\frac{1}{15}x^2 + \frac{2}{15}x + \frac{1}{5}\right) dx$ A1

$= \frac{19}{90}$ A1

[3]

Sección B

10. (a) (i) $a_1(t) = \frac{20-30}{2-0}$ M1A1

$a_1(t) = -5$ AG

(ii) $v_1(t) = -5t + 30$ A2

[4]

(b) La distancia total recorrida por la canica

$= \int_0^2 |v_1(t)| \mathrm{d}t$ (M1) por enfoque válido

$= \int_0^2 |-5t + 30| \mathrm{d}t$ (A1) por fórmula correcta

$= 50 \text{ cm}$ A1

[3]

(c) (i) $v_2(2) = 20$

$\therefore 20e^{b-0,2(2)} = 20$ M1

$e^{b-0,4} = 1$

$b - 0,4 = 0$ A1

$b = 0,4$ AG

(ii) $\int_2^c |v_2(t)| \mathrm{d}t = 50$

$\int_2^c 20e^{0,4-0,2t} \mathrm{d}t = 50$ (M1) por ecuación

Sea $u = 0,4 - 0,2t$
$\frac{\mathrm{d}u}{\mathrm{d}t} = -0,2 \Rightarrow -100\mathrm{d}u = 20\mathrm{d}t$
$t = c \Rightarrow u = 0,4 - 0,2c$
$t = 2 \Rightarrow u = 0,4 - 0,2(2) = 0$

(A1) por sustitución

$\int_0^{0,4-0,2c} -100e^u \mathrm{d}u = 50$ A1

$\left[-100e^u\right]_0^{0,4-0,2c} = 50$

$e^{0,4-0,2c} - e^0 = -0,5$ (M1) por sustitución

$e^{0,4-0,2c} = 0,5$

$0,4 - 0,2c = \ln 0,5$

$0,4 - \ln 0,5 = 0,2c$

$c = 5,465735903$

$c = 5,47$ A1

[7]

11. (a) Las coordenadas de A, B' y C son $(-3, 0, 0)$, $(0, 4, 0)$ y $(0, 0, 8)$ respectivamente. A1

$\mathbf{n} = \overrightarrow{\mathrm{AB}'} \times \overrightarrow{\mathrm{AC}}$ M1

$\mathbf{n} = (3\mathbf{i} + 4\mathbf{j}) \times (3\mathbf{i} + 8\mathbf{k})$ A1

$$\mathbf{n} = \begin{pmatrix} (4)(8) - (0)(0) \\ (0)(3) - (3)(8) \\ (3)(0) - (4)(3) \end{pmatrix}$$

$\mathbf{n} = 32\mathbf{i} - 24\mathbf{j} - 12\mathbf{k}$ A1

La ecuación cartesiana del plano π_2:

$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (32\mathbf{i} - 24\mathbf{j} - 12\mathbf{k})$
$= -3\mathbf{i} \cdot (32\mathbf{i} - 24\mathbf{j} - 12\mathbf{k})$ M1A1

$32x - 24y - 12z = (-3)(32) + (0)(-24) + (0)(-12)$

$32x - 24y - 12z = -96$

$8x - 6y - 3z = -24$ AG

[6]

(b) Las coordenadas de B son $(0, -4, 0)$. (A1) por valores correctos

El volumen de la pirámide ABCC'

$= \frac{1}{3}\left(\frac{(\mathrm{BB}')(\mathrm{OA})}{2}\right)(\mathrm{OC})$ (M1) por enfoque válido

$= \frac{1}{3}\left(\frac{(4-(-4))(0-(-3))}{2}\right)(8)$ A1

$= 32$ A1

[4]

(c) $\mathbf{n}_1 = 8\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$

$\mathbf{n}_2 = 8\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ (A1) por valores correctos

Sea θ el ángulo obtuso entre los planos.

$\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2|\cos\theta$ (M1) por enfoque válido

$(8)(8) + (6)(-6) + (-3)(-3)$

$= (\sqrt{8^2 + 6^2 + (-3)^2})(\sqrt{8^2 + (-6)^2 + (-3)^2})\cos\theta$ (A1) por sustitución

$37 = (\sqrt{109})(\sqrt{109})\cos\theta$

$\cos\theta = \frac{37}{109}$ A1

$\theta = 70{,}15665929^\circ$

El ángulo obtuso requerido

$= 180^\circ - 70{,}15665929^\circ$

$= 109{,}8433407^\circ$

$= 110^\circ$ A1

[5]

(d) El punto medio de BC

$= \left(\frac{0+0}{2}, \frac{-4+0}{2}, \frac{0+8}{2}\right)$

$= (0, -2, 4)$ (A1) por valores correctos

$\mathbf{n}_3 = \mathbf{n}_1 \times \mathbf{n}_2$ (M1) por enfoque válido

$\mathbf{n}_3 = (8\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) \times (8\mathbf{i} - 6\mathbf{j} - 3\mathbf{k})$

$$\mathbf{n}_3 = \begin{pmatrix} (6)(-3) - (-3)(-6) \\ (-3)(8) - (8)(-3) \\ (8)(-6) - (6)(8) \end{pmatrix}$$

$\mathbf{n}_3 = -36\mathbf{i} - 96\mathbf{k}$ A1

La ecuación vectorial de la recta:

$$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix} + t\begin{pmatrix} -36 \\ 0 \\ -96 \end{pmatrix}$$ A1

$$\begin{cases} x = -36t \\ y = -2 \\ z = 4 - 96t \end{cases}$$

$\frac{x}{-36} = \frac{z-4}{-96}, y = -2$ A1

[5]

12. (a) (i) $x^2\dfrac{dy}{dx}+6y=x^3e^{x^2+\frac{6}{x}}$

$\dfrac{dy}{dx}+\dfrac{6}{x^2}y=xe^{x^2+\frac{6}{x}}$ (A1) por enfoque correcto

El factor integrante

$=e^{\int\frac{6}{x^2}dx}$ (M1) por enfoque válido

$=e^{-\frac{6}{x}}$ A1

$\therefore e^{-\frac{6}{x}}\dfrac{dy}{dx}+e^{-\frac{6}{x}}\cdot\dfrac{6}{x^2}y=e^{-\frac{6}{x}}\cdot xe^{x^2+\frac{6}{x}}$ (M1) por enfoque válido

$e^{-\frac{6}{x}}\dfrac{dy}{dx}+e^{-\frac{6}{x}}\cdot\dfrac{6}{x^2}y=xe^{x^2}$

$\dfrac{d}{dx}\left(ye^{-\frac{6}{x}}\right)=xe^{x^2}$ (A1) por enfoque correcto

$ye^{-\frac{6}{x}}=\int xe^{x^2}dx$

Sea $u=x^2$. (M1) por sustitución

$\dfrac{du}{dx}=2x\Rightarrow\dfrac{1}{2}du=xdx$

$\therefore ye^{-\frac{6}{x}}=\int e^u\cdot\dfrac{1}{2}du$ (A1) por enfoque correcto

$ye^{-\frac{6}{x}}=\dfrac{1}{2}\int e^u du$

$ye^{-\frac{6}{x}}=\dfrac{1}{2}e^u+C$

$ye^{-\frac{6}{x}}=\dfrac{1}{2}e^{x^2}+C$ A1

$ye^{-\frac{6}{x}}=\dfrac{e^{x^2}+C}{2}$

$y=\dfrac{e^{\frac{6}{x}}(e^{x^2}+C)}{2}$ A1

$\dfrac{e^7}{2}=\dfrac{e^{\frac{6}{1}}(e^{1^2}+C)}{2}$ (M1) por sustitución

$\dfrac{e^7}{2}=\dfrac{e^7+Ce^6}{2}$

$Ce^6=0$

$C=0$

$\therefore y = \dfrac{e^{\frac{6}{x}+x^2}}{2}$ A1

(ii) $\dfrac{e^{11}}{2}$ A1

[12]

(b) (i) $\begin{cases} x_{n+1} = x_n + 0,1 \\ y_{n+1} = y_n + 0,1 \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x_n,\, y_n)} \end{cases}$ M1

$x_0 = 1,\ y_0 = \dfrac{e^7}{2}$ A1

$x_1 = 1 + 0,1$

$x_1 = 1,1$

$y_1 = \dfrac{e^7}{2} + 0,1\left(1e^{1^2+\frac{6}{1}} - \dfrac{6}{1^2}\left(\dfrac{e^7}{2}\right)\right)$ M1A1

$y_1 = \dfrac{e^7}{2} - 0,2e^7$

$y_1 = \dfrac{3e^7}{10}$ AG

(ii) $23435,5461$ A2

[6]

(c) $23435,5461 < \dfrac{e^{11}}{2}$ R1

[1]

Solución de Práctica de Prueba 3 de AE NS Set 4

1. (a) $F(2)$

$$=\sum_{r=1}^{2}\operatorname{sen}\frac{\pi}{2(2)}\operatorname{sen}\frac{r\pi}{2}$$ (M1) por sustitución

$$=\operatorname{sen}\frac{\pi}{4}\sum_{r=1}^{2}\operatorname{sen}\frac{r\pi}{2}$$

$$=\operatorname{sen}\frac{\pi}{4}\left(\operatorname{sen}\frac{\pi}{2}+\operatorname{sen}\pi\right)$$ A1

$$=\left(\operatorname{sen}\frac{\pi}{4}\right)(1+0)$$

$$=\operatorname{sen}\frac{\pi}{4}$$ A1

[3]

(b) (i) $\cos(x-y)-\cos(x+y)$

$=\cos x\cos y+\operatorname{sen}x\operatorname{sen}y$

$-(\cos x\cos y-\operatorname{sen}x\operatorname{sen}y)$ A2

$=2\operatorname{sen}x\operatorname{sen}y$ AG

(ii) Sean $x=\dfrac{A+B}{2}$ y $y=\dfrac{B-A}{2}$.

$\cos(x-y)-\cos(x+y)$

$$=\cos\left(\frac{A+B}{2}-\frac{B-A}{2}\right)$$

$$-\cos\left(\frac{A+B}{2}+\frac{B-A}{2}\right)$$ A1

$$=\cos\frac{2A}{2}-\cos\frac{2B}{2}$$ M1

$=\cos A-\cos B$

$$\therefore \cos A-\cos B=2\operatorname{sen}\frac{A+B}{2}\operatorname{sen}\frac{B-A}{2}$$ AG

[4]

(c) $F(4)$

$$= \sum_{r=1}^{4} \operatorname{sen} \frac{\pi}{2(4)} \operatorname{sen} \frac{r\pi}{4}$$

$$= \operatorname{sen} \frac{\pi}{8} \sum_{r=1}^{4} \operatorname{sen} \frac{r\pi}{4}$$

$$= \operatorname{sen} \frac{\pi}{8} \left(\operatorname{sen} \frac{\pi}{4} + \operatorname{sen} \frac{\pi}{2} + \operatorname{sen} \frac{3\pi}{4} + \operatorname{sen} \pi \right)$$ A1

$$= \operatorname{sen} \frac{\pi}{8} \operatorname{sen} \frac{\pi}{4} + \operatorname{sen} \frac{\pi}{8} \operatorname{sen} \frac{\pi}{2}$$

$$+ \operatorname{sen} \frac{\pi}{8} \operatorname{sen} \frac{3\pi}{4} + \operatorname{sen} \frac{\pi}{8} \operatorname{sen} \pi$$

$$= \frac{1}{2}\left(\cos\left(\frac{\pi}{8} - \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{8} + \frac{\pi}{4}\right) \right)$$

$$+ \frac{1}{2}\left(\cos\left(\frac{\pi}{8} - \frac{\pi}{2}\right) - \cos\left(\frac{\pi}{8} + \frac{\pi}{2}\right) \right)$$ M1A1

$$+ \frac{1}{2}\left(\cos\left(\frac{\pi}{8} - \frac{3\pi}{4}\right) - \cos\left(\frac{\pi}{8} + \frac{3\pi}{4}\right) \right)$$

$$= \frac{1}{2}\left(\begin{array}{l} \cos\left(-\frac{\pi}{8}\right) - \cos\frac{3\pi}{8} + \cos\left(-\frac{3\pi}{8}\right) - \cos\frac{5\pi}{8} \\ + \cos\left(-\frac{5\pi}{8}\right) - \cos\frac{7\pi}{8} \end{array} \right)$$

$$= \frac{1}{2}\left(\begin{array}{l} \cos\frac{\pi}{8} - \cos\frac{3\pi}{8} + \cos\frac{3\pi}{8} - \cos\frac{5\pi}{8} \\ + \cos\frac{5\pi}{8} - \cos\frac{7\pi}{8} \end{array} \right)$$ A1

$$= \frac{1}{2}\left(\cos\frac{\pi}{8} - \cos\frac{7\pi}{8} \right)$$

$$= \frac{1}{2}\left(2 \operatorname{sen} \frac{\frac{\pi}{8} + \frac{7\pi}{8}}{2} \operatorname{sen} \frac{\frac{7\pi}{8} - \frac{\pi}{8}}{2} \right)$$ A1

$$= \operatorname{sen} \frac{\pi}{2} \operatorname{sen} \frac{3\pi}{8}$$

$$= \operatorname{sen} \frac{3\pi}{8}$$ A1

[6]

(d) $F(n)$

$$= \sum_{r=1}^{n} \operatorname{sen}\frac{\pi}{2n}\operatorname{sen}\frac{r\pi}{n}$$

$$= \sum_{r=1}^{n} \frac{1}{2}\left(\cos\left(\frac{\pi}{2n}-\frac{r\pi}{n}\right)-\cos\left(\frac{\pi}{2n}+\frac{r\pi}{n}\right)\right)$$ M1A1

$$= \sum_{r=1}^{n} \frac{1}{2}\left(\cos\left(\frac{\pi}{2n}-\frac{2r\pi}{2n}\right)-\cos\left(\frac{\pi}{2n}+\frac{2r\pi}{2n}\right)\right)$$

$$= \sum_{r=1}^{n} \frac{1}{2}\left(\cos\frac{(1-2r)\pi}{2n}-\cos\frac{(1+2r)\pi}{2n}\right)$$ M1

$$= \frac{1}{2}\begin{pmatrix} \cos\frac{(1-2(1))\pi}{2n}-\cos\frac{(1+2(1))\pi}{2n} \\ +\cos\frac{(1-2(2))\pi}{2n}-\cos\frac{(1+2(2))\pi}{2n} \\ +\cdots+\cos\frac{(1-2n)\pi}{2n}-\cos\frac{(1+2n)\pi}{2n} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} \cos\left(-\frac{\pi}{2n}\right)-\cos\frac{3\pi}{2n}+\cos\left(-\frac{3\pi}{2n}\right)-\cos\frac{5\pi}{2n} \\ +\cdots+\cos\frac{(1-2n)\pi}{2n}-\cos\frac{(1+2n)\pi}{2n} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} \cos\frac{\pi}{2n}-\cos\frac{3\pi}{2n}+\cos\frac{3\pi}{2n}-\cos\frac{5\pi}{2n} \\ +\cdots+\cos\frac{(2n-1)\pi}{2n}-\cos\frac{(1+2n)\pi}{2n} \end{pmatrix}$$ A1

$$= \frac{1}{2}\left(\cos\frac{\pi}{2n}-\cos\frac{(1+2n)\pi}{2n}\right)$$ M1

$$= \frac{1}{2}\left(2\operatorname{sen}\frac{\frac{\pi}{2n}+\frac{(1+2n)\pi}{2n}}{2}\operatorname{sen}\frac{\frac{(1+2n)\pi}{2n}-\frac{\pi}{2n}}{2}\right)$$ A1

$$= \operatorname{sen}\frac{(2+2n)\pi}{4n}\operatorname{sen}\frac{2n\pi}{4n}$$

$$= \operatorname{sen}\frac{(1+n)\pi}{2n}\operatorname{sen}\frac{\pi}{2}$$

$$\therefore F(n) = \operatorname{sen}\frac{(1+n)\pi}{2n}$$ AG

[6]

(e) $\left|z^r - 1\right|$

$$= \left|\left(\cos\frac{2\pi}{n} + \mathrm{i}\,\mathrm{sen}\frac{2\pi}{n}\right)^r - 1\right|$$

$$= \left|\cos\frac{2\pi r}{n} + \mathrm{i}\,\mathrm{sen}\frac{2\pi r}{n} - 1\right|$$ (M1) por enfoque válido

$$= \sqrt{\left(\cos\frac{2\pi r}{n} - 1\right)^2 + \mathrm{sen}^2\frac{2\pi r}{n}}$$

$$= \sqrt{\cos^2\frac{2\pi r}{n} - 2\cos\frac{2\pi r}{n} + 1 + \mathrm{sen}^2\frac{2\pi r}{n}}$$ M1

$$= \sqrt{2 - 2\cos\frac{2\pi r}{n}}$$

$$= \sqrt{2 - 2\left(1 - 2\,\mathrm{sen}^2\frac{\pi r}{n}\right)}$$ A1

$$= \sqrt{4\,\mathrm{sen}^2\frac{\pi r}{n}}$$

$$= 2\,\mathrm{sen}\frac{\pi r}{n}$$

$$\because -1 \le \mathrm{sen}\frac{\pi r}{n} \le 1$$ R1

$$\therefore \left|z^r - 1\right| \le 2$$ A1

[5]

(f) $G(n)$

$= \sum_{r=1}^{n} \left| z^r - 1 \right|$

$= \sum_{r=1}^{n} 2 \operatorname{sen} \frac{\pi r}{n}$

$= \frac{2\sum_{r=1}^{n} \operatorname{sen} \frac{\pi}{2n} \operatorname{sen} \frac{\pi r}{n}}{\operatorname{sen} \frac{\pi}{2n}}$ M1

$= \frac{2F(n)}{\operatorname{sen} \frac{\pi}{2n}}$ A1

$= \frac{2 \operatorname{sen} \frac{(1+n)\pi}{2n}}{\operatorname{sen} \frac{\pi}{2n}}$

$= \frac{2 \cos\left(\frac{\pi}{2} - \frac{(1+n)\pi}{2n} \right)}{\operatorname{sen} \frac{\pi}{2n}}$ A1

$= \frac{2 \cos\left(\frac{n\pi}{2n} - \frac{\pi + n\pi}{2n} \right)}{\operatorname{sen} \frac{\pi}{2n}}$

$= \frac{2 \cos\left(-\frac{\pi}{2n} \right)}{\operatorname{sen} \frac{\pi}{2n}}$ M1

$= \frac{2 \cos \frac{\pi}{2n}}{\operatorname{sen} \frac{\pi}{2n}}$

$= 2 \cot \frac{\pi}{2n}$ A1

[5]

2. (a) (i) $I(0)$

$= \int_0^{\pi} x \mathrm{d}x$ M1

$= \left[\frac{1}{2}x^2\right]_0^{\pi}$ A1

$= \frac{1}{2}\pi^2 - \frac{1}{2}(0)^2$

$= \frac{1}{2}\pi^2$ AG

(ii) $I(1)$

$= \int_0^{\pi} x \operatorname{sen} x \mathrm{d}x$

Sea $\theta = \cos x$. (M1) por enfoque válido

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = -\operatorname{sen} x \Rightarrow (-1)\frac{\mathrm{d}\theta}{\mathrm{d}x} = \operatorname{sen} x$

$\therefore I(1)$

$= \int_0^{\pi} x(-1)\frac{\mathrm{d}(\cos x)}{\mathrm{d}x}\mathrm{d}x$

$= \left[-x\cos x\right]_0^{\pi} - \int_0^{\pi} \cos x \cdot \frac{\mathrm{d}(-x)}{\mathrm{d}x}\mathrm{d}x$ A1

$= \left[-x\cos x\right]_0^{\pi} - \int_0^{\pi} \cos x \cdot (-1)\mathrm{d}x$ A1

$= \left[-x\cos x\right]_0^{\pi} + \int_0^{\pi} \cos x \mathrm{d}x$

$= \left[-x\cos x\right]_0^{\pi} + \left[\operatorname{sen} x\right]_0^{\pi}$ A1

$= \left[-x\cos x + \operatorname{sen} x\right]_0^{\pi}$

$= (-\pi\cos\pi + \operatorname{sen}\pi) - (0 + \operatorname{sen} 0)$

$= \pi$ A1

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(b) (i) $I(n+2)$

$= \int_0^{\pi} x \operatorname{sen}^{n+2} x \mathrm{d}x$

$= \int_0^{\pi} x \operatorname{sen}^{n} x \operatorname{sen}^2 x \mathrm{d}x$ M1

$= \int_0^{\pi} x \operatorname{sen}^{n} x (1 - \cos^2 x) \mathrm{d}x$

$= \int_0^{\pi} x \operatorname{sen}^{n} x \mathrm{d}x - \int_0^{\pi} x \operatorname{sen}^{n} x \cos^2 x \mathrm{d}x$ A1

$= I(n) - \int_0^{\pi} x \operatorname{sen}^{n} x \cos^2 x \mathrm{d}x$ AG

(ii) $\int_0^{\pi} x\,\text{sen}^n x \cos^2 x\,\mathrm{d}x$

$$= \frac{1}{n+1}\int_0^{\pi} x\cos x \cdot \frac{\mathrm{d}(\text{sen}^{n+1}x)}{\mathrm{d}x}\mathrm{d}x$$

$$= \frac{1}{n+1}\left\{\begin{aligned}&\left[x\cos x\,\text{sen}^{n+1}x\right]_0^{\pi}\\&-\int_0^{\pi}\text{sen}^{n+1}x\cdot\frac{\mathrm{d}(x\cos x)}{\mathrm{d}x}\mathrm{d}x\end{aligned}\right\}$$ A1

$$= \frac{1}{n+1}\left\{\begin{aligned}&\left[x\cos x\,\text{sen}^{n+1}x\right]_0^{\pi}\\&-\int_0^{\pi}\text{sen}^{n+1}x(\cos x - x\,\text{sen}\,x)\mathrm{d}x\end{aligned}\right\}$$ A1

$$= \frac{1}{n+1}\left\{\begin{aligned}&\left[x\cos x\,\text{sen}^{n+1}x\right]_0^{\pi}\\&-\int_0^{\pi}(\text{sen}^{n+1}x\cos x - x\,\text{sen}^{n+2}x)\mathrm{d}x\end{aligned}\right\}$$

$$= \frac{1}{n+1}\left\{\begin{aligned}&\left[x\cos x\,\text{sen}^{n+1}x\right]_0^{\pi}\\&-\int_0^{\pi}\text{sen}^{n+1}x\cos x\mathrm{d}x + I(n+2)\end{aligned}\right\}$$ M1

$$= \frac{1}{n+1}\left\{\begin{aligned}&(\pi\cos\pi\,\text{sen}^{n+1}\pi - 0)\\&-\int_0^{\pi}\text{sen}^{n+1}x\cos x\mathrm{d}x + I(n+2)\end{aligned}\right\}$$ M1

$$= \frac{1}{n+1}\left\{-\int_0^{\pi}\text{sen}^{n+1}x\cos x\mathrm{d}x + I(n+2)\right\}$$

Sea $u = \text{sen}\,x$. (M1) por sustitución

$$\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x \Rightarrow \mathrm{d}u = \cos x\mathrm{d}x$$

$$x = \pi \Rightarrow u = \text{sen}\,\pi = 0$$

$$x = 0 \Rightarrow u = \text{sen}\,0 = 0$$

$$\therefore \int_0^{\pi} x\,\text{sen}^n x\cos^2 x\mathrm{d}x$$

$$= \frac{1}{n+1}\left\{-\int_0^{0} u^{n+1}\mathrm{d}u + I(n+2)\right\}$$ A1

$$= \frac{1}{n+1}(0 + I(n+2))$$

$$= \frac{1}{n+1}I(n+2)$$ A1

(iii) $I(n+2)=I(n)-\frac{1}{n+1}I(n+2)$ A1

$(n+1)I(n+2)=(n+1)I(n)-I(n+2)$

$(n+2)I(n+2)=(n+1)I(n)$ M1

$I(n+2)=\frac{n+1}{n+2}I(n)$ AG

[11]

(c) (i) $I(4)$

$=\frac{2+1}{2+2}I(2)$ M1

$=\frac{3}{4}\left(\frac{0+1}{0+2}I(0)\right)$ M1

$=\frac{3}{4}\left(\frac{1}{2}\cdot\frac{1}{2}\pi^2\right)$

$=\frac{3}{16}\pi^2$ A1

(ii) $I(7)$

$=\frac{5+1}{5+2}I(5)$ M1

$=\frac{6}{7}\left(\frac{3+1}{3+2}I(3)\right)$ M1

$=\frac{6}{7}\left(\frac{4}{5}\right)\left(\frac{1+1}{1+2}I(1)\right)$

$=\frac{6}{7}\left(\frac{4}{5}\right)\left(\frac{2}{3}\pi\right)$

$=\frac{16}{35}\pi$ A1

[6]

(d) $0\le \operatorname{sen} x\le 1$ for $0\le x\le \pi$. A1

Por lo tanto, $\operatorname{sen}^2 x\le \operatorname{sen} x\le 1$, implica que

$\int_0^\pi x\operatorname{sen}^{2n-2}x\cdot \operatorname{sen}^2 x\mathrm{d}x$

$\le \int_0^\pi x\operatorname{sen}^{2n-2}x\cdot \operatorname{sen} x\mathrm{d}x\le \int_0^\pi x\operatorname{sen}^{2n-2}x\cdot 1\mathrm{d}x$. R1

Por lo tanto, $I(2n)\le I(2n-1)\le I(2n-2)$ para $n\ge 1$. AG

[2]